

# Wave Functions, Evolution Equations and Evolution Kernels from Light-Ray Operators of QCD

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## Abstract

The widely used nonperturbative wave functions and distribution functions of QCD are determined as matrix elements of light-ray operators. These operators appear as large momentum limit of nonlocal hadron operators or as summed up local operators in light-cone expansions. Nonforward one-particle matrix elements of such operators lead to new distribution amplitudes describing both hadrons simultaneously. These distribution functions depend besides other variables on two scaling variables. They are applied for the description of exclusive virtual Compton scattering in the Bjorken region near forward direction and the two meson production process. The evolution equations for these distribution amplitudes are derived on the basis of the renormalization group equation of the considered operators. This includes that also the evolution kernels follow from the anomalous dimensions of these operators. Relations between different evolution kernels (especially the Altarelli-Parisi and the Brodsky-Lepage) kernels are derived and explicitly checked for the existing two-loop calculations of QCD. Technical basis of these results are support and analytically properties of the anomalous dimensions of light-ray operators obtained with the help of the  $\alpha$ -representation of Green's functions.

## I. INTRODUCTION

It is generally accepted that quantum chromodynamics (QCD) allows the theoretical description of scattering processes at large momentum transfer. Thereby, it is possible to calculate the so-called hard scattering subprocess perturbatively, and for the remaining parts of the process one uses "parton distribution functions" [1] or "wave functions" [2]. These functions are not calculable perturbatively, but have to be determined phenomenologically. Nevertheless additional information on them can be obtained because they satisfy evolution equations.

In this paper we take into account that the usually applied nonperturbative distribution and wave functions are directly related to special matrix elements of (nonlocal) light-ray operators [2–5]. The parton distribution amplitude (corresponding to the forward matrix element) is necessary for the description of inclusive lepton-hadron scattering and the meson (hadron) wave functions (corresponding to the matrix element between the one-hadron and the vacuum state) is needed for the treatment of exclusive large transverse momentum processes. Here we discuss the question what role plays the general one-particle matrix element between hadrons of different momenta. To these matrix elements belong new distribution amplitudes describing both hadrons simultaneously. Such nonperturbative distribution amplitudes are necessary if there is no large momentum transfer between the hadrons (otherwise we could use a hard scattering part and a separate description of each hadron by a non-perturbative function). As an example we show that these functions can be applied to the prescription of the virtual Compton scattering amplitude in the Bjorken region near forward scattering or to two meson production processes. Thereby it is extremely interesting that we obtain a very simple representation of the Compton scattering amplitude in leading order. In fact, the virtual Compton amplitude can be represented as a convolution of a hard scattering part with the distribution amplitude, so that the analogy to the simple parton picture remains.

A second topic of this paper is the investigation of the evolution equations and evolution kernels for these nonperturbative distribution amplitudes. Here we take into account that all these amplitudes are matrix elements of light-ray operators, satisfying renormalization group equations. Therefore, the evolution equations are basically renormalization group equations. Of course, by forming matrix elements the renormalization group equations for light-ray operators turn over to evolution equations for these matrix elements. Using such a general procedure, we are able (in principle) to derive evolution equations for all matrix elements of such operators. The Brodsky-Lepage equation [2] for the meson wave function and the Altarelli-Parisi equation [1] for the quark distribution function of deep inelastic scattering appear as special cases. It is important that the evolution kernels are functionals of the anomalous dimension of the considered operator. A trivial but interesting point is that thereby the range of the variables may change according to the support properties of the distribution amplitude. A very general kernel which is in fact the complete anomalous dimension of the underlying light-ray operator is the extended Brodsky-Lepage (BL) kernel. It represents a partly diagonalized kernel generating all one-parametric evolution equations. In the one-loop approximation, a complete diagonalized form of such a kernel following from nonlocal (conformal) operator product expansions has been studied in Ref. [6].

Besides of these general considerations we discuss also some interesting technical prob-

lems concerning the explicit determination of these kernels. In QCD, the BL-kernel is calculated as an evolution kernel in a restricted range of the variables. The interesting question is: Exists there a possibility to determine the extended BL-kernel knowing this restricted kernel only? We prove that the partly Fourier transformed BL-kernel has holomorphic properties allowing such an extension. In the following these results are applied to the one- and two-loop calculations of the BL-kernel in QCD. Without additional diagrammatic calculations we are able to determine the extended BL-kernel [7] of QCD. Finally, we perform the limiting procedure leading from the extended BL-kernel to the Altarelli-Parisi (AP) kernel, and compare the resulting expression with the directly calculated results [8–10]. In this way we obtain a consistency check for the very complicated two-loop calculations [9,10]. Taking into consideration the recent check [11] based on conformal Wardidentities and consistence equations, we conclude that the BL-kernel has been correctly computed.

All our investigations concern the flavour nonsinglet operators. Investigations of singlet operators are in principle possible, however technically more complicated.

It remains to give some remarks concerning the light-ray operators and its anomalous dimensions. Light-ray operators were first introduced by O. I. Zavialov [12] in connection with his proof of the light-cone operator product expansion. In opposite to the standard formulation [13] he did not use the expansion in terms of local operators [13] but introduced light-ray operators which are in fact summed up local operators. We also use this technique for the treatment of the virtual Compton scattering amplitude. We hope to convince the reader that this method is very effective. An independent approach based on the same ideas was developed and applied from the Leningrad group [6,14]. Another source for the introduction of light-ray operators are the speculations about hadron operators in QCD [15]. Hadrons are composed of quarks and gluons, therefore hadron operators have to be bi- or trilocal operators containing the basic fields of QCD. The postulate of gauge invariance leads to string operators finally. If we want to apply these operators in hard scattering processes then for large internal momenta these operators turn over to the light-ray operators considered here. It is now clear that for example bilocal operators where the distance between both positions of the involved field operators is light-like possess additional divergences. Therefore they need a special renormalization procedure which leads to more complicated anomalous dimensions as usually discussed.

To avoid complicated notations in the main text we generally write down renormalized expressions. Only in the Appendix we also use unrenormalized expressions. An important technical task is the investigation of the anomalous dimensions of these operators, especially the correct determination of its domain of definition. Using the  $\alpha$ -representation of Feynman graphs, we first study this problem for the scalar theory and afterwards for QCD in axial gauge. For the last case we modify the  $\alpha$ -representation to include the more complicated gluon propagator of QCD in light-cone gauge.

The paper is organized as follows: In the second section we introduce the generalized distribution amplitudes as matrix elements of light-ray operators. In the next section we apply these distribution amplitudes for the prescription of the exclusive virtual Compton scattering and the two meson production by photons. In the fourth section we derive the evolution equations for these amplitudes. Problems concerning explicit expressions (two-loop calculations) and relation between different evolution kernels are discussed in the last section. Important technical details concerning the domain of definition of the anomalous dimension,

the perturbative calculation of hard scattering parts and other problems are shifted to Appendices. The Appendices are needed for the proofs only, the reader uninterested in technical details could skip them.

## II. NONPERTURBATIVE INPUTS: DISTRIBUTION AMPLITUDES

Up to now the application of nonperturbatively determined "parton distribution functions" and "wave functions" is an unavoidable part of perturbative QCD calculations. We start our considerations with the introduction of more general distribution amplitudes corresponding to nonforward scattering processes.

In the QCD picture hadrons are built up from quarks and gluons. So possible hadron operators are nonlocal operators containing quark and gluon fields. Typical meson operators are composed of quark and antiquark operators

$$O^a(x_1, x_2) = : \bar{\psi}(x_1) \Gamma \lambda^a U(x_1, x_2) \psi(x_2) : . \quad (2.1)$$

The path ordered phase factor,

$$U(x_1, x_2) = \mathcal{P} \exp \left\{ -ig \int_{x_2}^{x_1} dx_\mu A^\mu(x) \right\} \quad (2.2)$$

ensures the gauge invariance of the considered operator.  $A^\mu = A_c^\mu t^c$  denotes the gluon field,  $t^c$  are the generators of the colour group,  $\psi$  is the quark field,  $\Gamma$  symbolizes the necessary spin structure and  $\lambda^a$  is a generator of the flavour group corresponding to the considered meson. At large internal quark momenta the operator (2.1) turns over to an asymptotic operator defined on a straightline light-like path

$$O^a(\kappa_+, \kappa_-; \tilde{n}) = : \bar{\psi}(\kappa_1 \tilde{n}) (\tilde{n} \gamma) \lambda^a U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n}) \psi(\kappa_2 \tilde{n}) : , \quad \kappa_\pm = (\kappa_2 \pm \kappa_1)/2, \quad (2.3)$$

where

$$U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n}) = \mathcal{P} \exp \left\{ -ig \int_{\kappa_2}^{\kappa_1} d\tau \tilde{n} A(\tau \tilde{n}) \right\}.$$

The vector  $\tilde{n}$ , with  $\tilde{n}^2 = 0$ , defines the light-ray pointing into the direction of the large momentum flow of this process.

For the description of mesons involved in hard QCD processes one needs an "asymptotic" wave function as nonperturbative input. This meson wave function  $\Phi^a(x, Q^2)$  depending on the distribution parameter  $x$  ( $0 \leq x \leq 1$ ) and the momentum transfer  $Q^2$  can be defined as the expectation value of a nonlocal (light-ray) operator lying on the light-cone [2-4]:

$$\Phi^a(x = (1+t)/2, Q^2) = \int \frac{d\kappa_- |\tilde{n}P|}{2\pi} e^{i\kappa_- (\tilde{n}P)t} \frac{1}{\tilde{n}P} < 0 | O^a(\kappa_-; \tilde{n}) | P > | \mu^2 = Q^2, \quad (2.4)$$

where

$$O^a(\kappa_-; \tilde{n}) = O^a(\kappa_+ = 0, \kappa_-; \tilde{n}) = : \bar{\psi}(-\kappa_- \tilde{n}) (\tilde{n} \gamma) \lambda^a U(-\kappa_- \tilde{n}, \kappa_- \tilde{n}) \psi(\kappa_- \tilde{n}) : . \quad (2.5)$$

Here  $|P>$  denotes the one-particle state of a scalar meson of momentum  $P$  and  $O^a(\kappa_-; \tilde{n})$  is the light-ray operator with the same flavour content. As renormalization point we choose the

typical large momentum  $Q$  of the basic process to which the wave function contributes. The factor  $1/(\tilde{n}P)$  was introduced to compensate the  $\tilde{n}$ -dependence of the factor  $\tilde{n}\gamma$  in the operator  $O^a(\kappa_-; \tilde{n})$ . Contrary to the definition in Ref. [2] our expression (2.4) is gauge invariant and Lorentz-covariant. In an infinite momentum frame with  $P = (|P_z|, 0, 0, P_z)$  (neglecting the masses of the particles) and  $\tilde{n} = (1, 0, 0, -1)$  the distribution parameter  $x$  can be interpreted as usually:  $xP$  is the fraction of the meson momentum  $P$  which is carried by the quark. Then the mesonwave function is the probability amplitude for finding a quark-antiquark pair in the meson in dependence of the momentum fraction parameter  $x$  and the momentum transfer  $Q^2$ .

As next, we define the quark distribution functions  $q_i(z, Q^2)$  [1] with the distribution parameter  $z$ . These functions are necessary for the description of deep inelastic scattering. At first such functions have been introduced phenomenologically, later on as Mellin transforms of moments of forward matrix elements of local operators [1]. If we take into account that the operator

$$O_i(\kappa_+, \kappa_-) = : \bar{\psi}_i(\kappa_1 \tilde{n})(\tilde{n}\gamma)U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n})\psi_i(\kappa_2 \tilde{n}): \quad (2.6)$$

is the "generating operator" for all local operators,

$$((\tilde{n}D)^{n_1} \bar{\psi}_i(0))(\tilde{n}\gamma)(\tilde{n}D)^{n_2} \psi_i(0) = \frac{\partial^{n_1}}{\partial \kappa_1^{n_1}} \frac{\partial^{n_2}}{\partial \kappa_2^{n_2}} O_i(\kappa_+, \kappa_-)|_{\kappa_1=\kappa_2=0}, \quad (2.7)$$

then a definition of the quark distribution function as Fourier transform of the one-particle matrix element of the light-ray operator [5],

$$O_i(\kappa_-; \tilde{n}) = : \bar{\psi}_i(-\kappa_- \tilde{n})(\tilde{n}\gamma)U(-\kappa_- \tilde{n}, \kappa_- \tilde{n})\psi_i(\kappa_- \tilde{n}):, \quad (2.8)$$

seems to be understandable. Here  $\psi_i$  denotes the quark field with flavour component  $i$ . However, group theoretically this operator has no definite flavour content which leads to mixing problems with the flavour singlet gluon operator. To avoid this problem we consider here the flavour nonsinglet distribution function only:

$$q^a(z, Q^2) = \int \frac{d\kappa_- |\tilde{n}P|}{2\pi} e^{2i\kappa_- (\tilde{n}P)z} \frac{1}{\tilde{n}P} < P | O^a(\kappa_-; \tilde{n}) | P > |_{\mu^2=Q^2}. \quad (2.9)$$

Physically this distribution function describes the probability of finding internal quarks with the momentum fraction  $zP$ , where  $P$  is the external hadron momentum. If the matrix  $\lambda^a$  is chosen diagonal, then for positive distribution parameter  $z$  this function represents a linear combination of the quark distribution functions, and the extension for  $z < 0$  can be interpreted as a linear combination of the antiquark distribution functions.

Both functions in (2.4) and (2.9) are expectation values of the same operator used, however, for different physical processes and having different interpretations.

If we look at the definition (2.9) of the quark distribution function given as Fourier transform of a forward matrix element of a light-ray operator then the question arises: What is the physical meaning of nonforward matrix elements? As an example we study in the next section two-photon processes in the Bjorken region. The idea is that the product of the two electromagnetic currents near light-like distances can be expressed with help of

coefficient functions and the light-ray operators  $O^a(\kappa_-; \tilde{x})$ . (This generalizes the standard local operator product expansion).

If we consider the virtual Compton scattering amplitude in nonforward direction, than we have to form nonforward matrix elements of these light-ray operators (see the next section). This leads us directly to the following distribution amplitude,

$$q^a(t, \tau, \mu^2) = \int \frac{d\kappa_- |\tilde{n}P_+|}{2\pi} e^{i\kappa_- (\tilde{n}P_+)t} \frac{1}{\tilde{n}P_+} \langle P_2 | O^a(\kappa_-; \tilde{n}) | P_1 \rangle |_{\tilde{n}P_- = \tau \tilde{n}P_+}, \quad (2.10)$$

with  $P_\pm = P_2 \pm P_1$ . This function depends on the distribution parameter  $t$  and the quotient  $\tau = \tilde{n}P_- / \tilde{n}P_+$  of the projection of momenta  $P_\pm$  onto a light-like direction  $\tilde{n}$ , on the renormalization point  $\mu$  and the scalar products of the external momenta  $P_i P_j$ . For physical states  $|P_1\rangle$  and  $|P_2\rangle$  the additional variable  $\tau$  is restricted by

$$|\tau| = \left| \frac{\tilde{n}P_-}{\tilde{n}P_+} \right| = \left| \frac{P_-^0 - P_-^\parallel}{P_+^0 - P_+^\parallel} \right| \leq 1, \quad P^\parallel = \frac{\vec{n} \vec{P}}{|\vec{n}|}. \quad (2.11)$$

With the help of the  $\alpha$ -representation it is possible to investigate the support properties of the function  $q^a(t, \tau, \mu^2)$  with respect to the variable  $t$ . Extending the proof given in Ref. [4] for  $\tau = 1$  to arbitrary values of  $\tau$ , it turns out that

$$q^a(t, \tau, Q^2) = 0 \quad \text{for} \quad |t| > 1. \quad (2.12)$$

The distribution amplitude  $q^a(t, \tau, Q^2)$  is especially suited for the description of nonforward processes near to the forward case. Namely, if there is no large momentum transfer between the two hadrons, then it is impossible to describe the hadrons by individual wave functions and a hard scattering part, i.e. the parton picture with two separate wave functions cannot be applied. Here one has to apply one function describing both hadrons. An example of such a process will be discussed in the next section (it is interesting that a similar function was already proposed by Ref. [17]).

On the other hand, it is possible to consider the nonforward distribution amplitude (2.10) as an interpolating function between the quark distribution function (2.9) (for  $P_1 = P_2$ ) and the meson wave function (2.4), i.e. in the limit  $P_2 \rightarrow 0$  too. Of course, the state obtained by  $P_2 \rightarrow 0$  is not the vacuum state, but for mathematical considerations concerning the connection of evolution equations for forward and nonforward processes this is very useful. As result in section V we obtain relations between different evolution kernels (especially between the Brodsky-Lepage kernel and the Altarelli-Parisi kernel) which are highly nontrivial.

Therefore, we can perform two limits

$$\begin{aligned} \Phi^a(x = (1+t)/2, Q^2) &= \lim_{\tau \rightarrow -1} q^a(t, \tau, Q^2) && \text{meson wave function (formally!),} \\ q^a(z, Q^2) &= \lim_{\tau \rightarrow 0} q^a(z, \tau, Q^2) && \text{quark distribution function (formally!).} \end{aligned} \quad (2.13)$$

In short we add some further properties of the distribution amplitude (2.10):

- Because of the conjugation properties of the light-ray operators  $(O^a(\kappa_-; \tilde{n}))^+ = O^a(-\kappa_-; \tilde{n})$  it satisfies

$$(q^a(t, \tau, \mu^2))^* = q^a(t, -\tau, \mu^2). \quad (2.14)$$

This means that this amplitude is real for  $P_1 = P_2$ .

- The normalization of this distribution amplitude follows from the definition (2.10) and (2.5),

$$\int dt q^a(t, \tau, \mu^2) = (\tilde{n}P_+)^{-1} \tilde{n}^\nu < P_2 | J_\nu^a(0) | P_1 > |_{\tilde{n}P_- = \tau \tilde{n}P_+}, \quad (2.15)$$

where  $J_\nu^a(0) = : \bar{\psi}(0) \gamma_\nu \lambda^a \psi(0) :$  is a current with the flavour content  $a$ .

At last we give a representation of the generalized distribution amplitude with the help of a "spectral" function which is helpful for later calculations. Taking into account the ideas of the general Jost-Lehmann representation for  $T$ -amplitudes of two local operators (our operators (2.5) are in fact bilocal if we use the light-cone gauge in QCD) then the spectral functions have a finite support with respect to the new distribution variables  $z_+$  and  $z_-$ . In this way the matrix elements of light-ray operators can be expressed by "spectral" functions  $f^a(z_+, z_-, \mu^2)$  as follows

$$\frac{1}{\tilde{n}P_+} < P_2 | O^a(\kappa_-; \tilde{n}) | P_1 > = \int \int dz_+ dz_- e^{-i\kappa_- (\tilde{n}P_-) z_- - i\kappa_- (\tilde{n}P_+) z_+} f^a(z_+, z_-, \mu^2). \quad (2.16)$$

If we insert now this equation (2.16) into the definition of the distribution function (2.10), we obtain a new representation

$$\begin{aligned} q^a(t, \tau, Q^2) &= \int \int dz_+ dz_- \int \frac{d\kappa_- |\tilde{n}P_+|}{2\pi} e^{i\kappa_- (\tilde{n}P_+) (t - z_+ - \tau z_-)} f^a(z_+, z_-, Q^2) \\ &= \int dz_- f^a(z_+ = t - \tau z_-, z_-, Q^2), \end{aligned} \quad (2.17)$$

which shows in which way the mathematically independent distribution variables  $z_+$  and  $z_-$  turn over to the more physical variables  $t$  and  $\tau$ .

### III. STRUCTURE FUNCTIONS FOR TWO-PHOTON PROCESSES

One of the first crucial test of perturbative QCD was the investigation of the deep inelastic electron proton scattering in the Bjorken region. Theoretically it implies the knowledge of the imaginary part of the virtual Compton scattering amplitude. The first QCD treatments of this process relied on the light-cone operator product expansion and its relation to the moments of the structure functions [18].

Another important class of processes are exclusive scattering processes of hadrons at large momentum transfer [2]. Perturbative QCD calculations demand the introduction of wave functions describing the incoming and outgoing hadrons. The perturbation theory itself is

applicable to a hard scattering part which connects these wave functions. The justification of perturbation theory is related to the presence of large momenta. Whereas in the case of deep inelastic scattering the scattering amplitude is dominated by the contributions from the light-cone singularities in the  $x$ -space, here it must not be the case. Therefore its perturbative treatment in QCD can be quite different and of course the kinematical regions of both processes are also different.

Here we want to investigate the complete Compton scattering amplitude in the Bjorken region near forward scattering [19]. Note that usually perturbative considerations of exclusive Compton scattering amplitudes are performed in the fixed angle region - were experimental data are present or expected [20]. If we consider the Compton amplitude in a generalized Bjorken region then experimental investigations with present possibilities are very hard to realize. On the other hand from a theoretical point of view it embeds the Compton amplitude for forward scattering and is therefore interesting in itself. Because of the smallness of the momentum transfer between both hadrons it is not possible to introduce separately nonperturbative amplitudes (or wave functions) for each hadron. Surprisingly the treatment of this process leads in a very natural way to the introduction of new distribution amplitudes which for the limiting case of forward scattering turn over into the well-known parton distribution functions of deep inelastic scattering. Furthermore these new distribution amplitudes satisfy new evolution equations [7] which turn over to the Altarelli-Parisi equation [1] for the case of forward scattering.

To show the virtues of this construction we study additionally the two meson production by two photons [21]. For small momentum transfer between both mesons also here we introduce one distribution amplitude describing both mesons.

### A. Exclusive Virtual Compton Scattering

Here we consider the virtual nonforward Compton scattering amplitude of a spinless particle near forward direction but at a large off-shell momentum of the the incoming photon:

$$T_{\mu\nu}(P_+, P_-, q) = i \int d^4x e^{iqx} \langle P_2 | T \left( J_\mu \left( \frac{x}{2} \right) J_\nu \left( \frac{-x}{2} \right) \right) | P_1 \rangle, \quad (3.1)$$

where  $J_\mu(x) = (1/2) : \bar{\psi}(x) \gamma_\mu (\lambda^3 - \lambda^8/\sqrt{3}) \psi(x) :$  is the electromagnetic current of the hadrons (for flavour  $SU(3)$ ).

The considered kinematics defines a generalized Bjorken region where, in close analogy to deep inelastic scattering, this process is dominated by contributions from the light-cone.

$$\gamma^*(q_1) + H(P_1) = \gamma^*(q_2) + H(P_2), \quad (3.2)$$

$$\begin{aligned} P_+ &= P_1 + P_2 = (2E, \vec{0}), & E &= \sqrt{(m^2 + \vec{p}^2)}, \\ P_- &= P_2 - P_1 = q_1 - q_2 = (0, -2\vec{p}), & q &= (1/2)(q_1 + q_2). \end{aligned} \quad (3.3)$$

The last expressions inside the brackets denote the values of the momenta in the Breit frame, respectively. The generalized Bjorken region is given by

$$\nu = P_+ q = 2E q_0 \rightarrow \infty, \quad Q^2 = -q^2 \rightarrow \infty, \quad (3.4)$$

with the scaling variables

$$\xi = \frac{-q^2}{P_+q}, \quad \eta = \frac{P_-q}{P_+q} = \frac{q_1^2 - q_2^2}{2\nu}. \quad (3.5)$$

To understand this process better we introduce the angle between the vectors  $\vec{p}$  and  $\vec{q}$  in the Breit frame by  $\cos \phi = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| |\vec{q}|}$ . In terms of these variables we get

$$q_1^2 \approx -\left(\xi - \frac{|\vec{p}|}{E} \cos \phi\right) \nu, \quad q_2^2 \approx -\left(\xi + \frac{|\vec{p}|}{E} \cos \phi\right) \nu, \quad \eta \approx \frac{|\vec{p}|}{E} \cos \phi. \quad (3.6)$$

In opposite to deep inelastic scattering the variable  $\xi$  is not restricted to  $0 \leq \xi \leq 1$ ; for example  $q_2^2 = 0$  demands  $\xi \approx -(|\vec{p}|/E) \cos \phi$ .

In Appendix A we show that in this region the helicity amplitudes  $T(\lambda', \lambda) = \varepsilon_2^\mu(\lambda') T_{\mu\nu} \varepsilon_1^\nu(\lambda)$  asymptotically are given by

$$T(\lambda', \lambda) = (1/2) \varepsilon_2^\nu(\lambda') \varepsilon_{1\nu}(\lambda) T_\mu^\mu \quad (3.7)$$

for the transverse helicities, and vanish otherwise. Therefore, only the trace of the scattering amplitude has to be considered.

The first treatment of the Compton scattering amplitude with the help of a (local) light-cone operator product expansion [13,18], symbolized by

$$J(x) J(0) = \sum_{n=0}^{\infty} C_n(x^2) x^{\mu_1} \dots x^{\mu_n} O_{\mu_1 \dots \mu_n}(0),$$

has the serious disadvantage that the amplitude has to be reconstructed from an infinite sum. This may be accepted for deep inelastic scattering where the famous connection between the moments of the structure functions and the expectation values of the local light-cone operators exists, but in general it is unsatisfactory. The application of the nonlocal light-cone expansion [4,6,12,16,22] is a suitable possibility to overcome this drawback. In our special case it reads (in leading order):

$$J^\mu\left(\frac{x}{2}\right) J_\mu\left(\frac{-x}{2}\right) \approx \int d\kappa_+ d\kappa_- F_a(x^2, \kappa_+, \kappa_-; \mu^2) O^a(\kappa_+, \kappa_-; \tilde{x})_{(\mu^2)} \quad (3.8)$$

with the light-ray operators given in (2.3). The light-like vector

$$\tilde{x}(x, \rho) = x + \rho \frac{x\rho}{\rho^2} \left( \sqrt{1 - \frac{x^2 \rho^2}{(x\rho)^2}} - 1 \right)$$

is determined by  $x$  and parameterized by a fixed constant vector  $\rho$ . The singular coefficient functions  $F_a$  are determined perturbatively; in the Born approximation they are given by

$$F_a(x^2, \kappa_+, \kappa_-) \approx i e_a \left( 2\pi^2 (x^2 - i\epsilon)^2 \right)^{-1} \delta(\kappa_+) (\delta(\kappa_- - 1/2) - \delta(\kappa_- + 1/2)), \quad (3.9)$$

with  $e_a = (2/9)\delta_{a0} + (1/6)\delta_{a3} + (1/6\sqrt{3})\delta_{a8}$ . Possible contact terms arising at the point  $x = 0$  lead to trivial contributions to the real part of the scattering amplitude and are suppressed.

Inserting the light-cone expansion (3.8) into the trace of the scattering amplitude (3.1) and using the representation (2.16) for the matrix elements  $\langle P_2|O^a(\kappa_-; \tilde{x})|P_1\rangle$ ,

$$\langle P_2|O^a(\kappa_-; \tilde{x})|P_1\rangle = \tilde{x}P_+ \int \int dz_+ dz_- e^{-i\kappa_-(\tilde{x}P_-)z_- - i\kappa_-(\tilde{x}P_+)z_+} f^a(z_+, z_-; P_i P_j, \mu^2),$$

we get

$$T_\mu^\mu(P_+, P_-, q) \approx 2 \int \int dz_+ dz_- \tilde{F}^a(\xi, \eta; z_+, z_-) f^a(z_+, z_-; P_i P_j, \mu^2 = Q^2), \quad (3.10)$$

with

$$\tilde{F}^a = -e^a \int \frac{d^4 x}{2\pi^2} \int d\kappa_- e^{ix[q-(P_+z_+ + P_-z_-)\kappa_-]} \frac{P_+ x}{(x^2 - i\epsilon)^2} (\delta(\kappa_- - 1/2) - \delta(\kappa_- + 1/2)).$$

(Here the approximation  $\tilde{x} \approx x$  has been used.)

To obtain a useful expression in momentum space the Fourier transform has to be carried out. Using

$$\int \frac{d^4 x}{2\pi^2} e^{ix[q-k]} \frac{P_+ x}{(x^2 - i\epsilon)^2} = \frac{P_+(q-k)}{(q-k)^2 + i\epsilon} \approx \frac{P_+ q}{q^2 - 2qk + i\epsilon}, \quad k = (P_+ z_+ + P_- z_-) \kappa_- \quad (3.11)$$

we find

$$T_\mu^\mu \approx 2 \int \int dz_+ dz_- \left( \frac{1}{\xi + z_+ + \eta z_-} - \frac{1}{\xi - z_+ - \eta z_-} \right) e^a f^a(z_+, z_-; P_i P_j, \mu^2 = Q^2). \quad (3.12)$$

In this form the result is unsatisfactory. It depends explicitly on the two distribution variables  $z_+$ ,  $z_-$  of the distribution amplitude  $f^a(z_+, z_-)$ . However, it is possible to simplify this expression further, and to derive an expression for the  $T$ -amplitude containing only one integral over the standard distribution amplitude. For this purpose we substitute  $z_+ = t - \eta z_-$  so that

$$\begin{aligned} T_\mu^\mu &\approx 2 \int dt \left( \frac{1}{\xi + t} - \frac{1}{\xi - t} \right) \int dz_- e^a f^a(z_+ = t - \eta z_-, z_-; P_i P_j, \mu^2 = Q^2) \\ &\approx 2 \int dt \left( \frac{1}{\xi + t} - \frac{1}{\xi - t} \right) e^a q^a(t, \eta; P_i P_j; \mu^2 = Q^2), \end{aligned} \quad (3.13)$$

where we have used the definition (2.17) with  $\tau$  replaced by  $\eta$ ,

$$q^a(t, \eta; P_i P_j, Q^2) = \int dz_- f^a(z_+ = t - \eta z_-, z_-; P_i P_j, Q^2).$$

Taking into account Eq. (3.7), we obtain the final result

$$T(\lambda', \lambda) \approx \varepsilon_2^\mu(\lambda') \varepsilon_{1\mu}(\lambda) \int dt \left( \frac{1}{\xi + t} - \frac{1}{\xi - t} \right) e^a q^a(t, \eta; P_i P_j; \mu^2 = Q^2). \quad (3.14)$$

Note that in order to obtain Eq. (3.14) it was necessary to turn over from the non-perturbative function  $f^a(z_+, z_-)$  back to the distribution amplitude  $q^a(t, \eta)$ , however, with changed variables. The original definition (2.10) of  $q^a(t, \tau)$  contains the light-like vector  $\tilde{n}$  which would be given here by  $\tilde{x}$ . This would lead to another definition of the variable  $\tau$  by  $\tau = \tilde{x}P_-/\tilde{x}P_+$  which does not make sense. But instead of this unphysical variable here the physical variable  $\eta = qP_-/qP_+$  appears. This amplitude generalizes the parton distribution function of deep inelastic scattering with the above mentioned properties. Note that  $q^a(t, Q^2) = q^a(t, \eta = 0, Q^2)$  is essentially the parton distribution function.

According to the optical theorem the absorptive part of the scattering amplitude  $W_{\mu\nu} = \text{Im}T_{\mu\nu}/(2\pi)$  is related to the total cross section in deep inelastic scattering. Since  $q^a(t, \eta = 0, Q^2)$  is real from equation (3.13) we get directly

$$\begin{aligned} W_\mu^\mu &\approx - \int_{-1}^1 dt (\delta(\xi - t) - \delta(\xi + t)) e_a q^a(t, \eta = 0; \mu^2 = Q^2) \\ &\approx - (e_a q^a(\xi, \eta = 0; \mu^2 = Q^2) - e_a q^a(-\xi, \eta = 0; \mu^2 = Q^2)). \end{aligned} \quad (3.15)$$

Here

$$- q^a(-\xi, \eta = 0; \mu^2 = Q^2) = \bar{q}^a(\xi, \eta = 0; \mu^2 = Q^2) \quad (3.16)$$

represents the antiquark distribution amplitude.

## B. Two Meson Production

As a further instructive example we discuss the two meson production by two photons

$$\gamma^*(q_1) + \gamma^*(q_2) = M(P_1) + M(P_2). \quad (3.17)$$

This is a crossed process with respect to the virtual Compton scattering. As variables we choose

$$q = (1/2)(q_1 - q_2), \quad P_+ = P_1 + P_2, \quad P_- = P_2 - P_1. \quad (3.18)$$

Again the generalized Bjorken region is defined by

$$\nu = P_+ q = 2E q_0 \rightarrow \infty, \quad Q^2 = -q^2 \rightarrow \infty, \quad (3.19)$$

with the scaling variables

$$\xi = \frac{-q^2}{P_+ q}, \quad \eta = \frac{P_- q}{P_+ q}. \quad (3.20)$$

The virtual photons are taken at large off shell momenta  $q_1^2 \rightarrow -\infty, q_2^2 \rightarrow -\infty$ . Then the range of the scaling variable is  $|\xi| \geq 1$ . In the center of mass system with fixed energy of the two mesons these scaling variables can be expressed as

$$\xi = \frac{-q^2}{P_+ q} = -\frac{q_1^2 + q_2^2}{q_1^2 - q_2^2}, \quad \eta = \frac{P_- q}{P_+ q} \approx \frac{\vec{p} \cdot \vec{q}}{E q_0} \approx \cos \phi. \quad (3.21)$$

In the leading approximation the scattering amplitude can be written as [see (3.7)]

$$T(P_+, P_-, q; \lambda', \lambda) = \frac{1}{2} \varepsilon_2^\nu(\lambda') \varepsilon_{1\nu}(\lambda) i \int d^4x e^{iqx} \langle P_1, P_2 | T \left( J_\mu \left( \frac{x}{2} \right) J^\mu \left( \frac{-x}{2} \right) \right) | 0 \rangle. \quad (3.22)$$

After the application of the operator product expansion, and the calculation of the coefficient function we arrive at the same type of expressions as (3.13)

$$T_\mu^\mu(P_+, P_-, q) \approx 2 \int dt \left( \frac{1}{\xi + t} - \frac{1}{\xi - t} \right) e_a q_m^a(t, \eta; \mu^2 = Q^2), \quad (3.23)$$

where the distribution amplitudes reads now

$$q_m^a(t, \eta; \mu^2) = \int \frac{d\kappa_- |\tilde{x}P_+|}{2\pi(\tilde{x}P_+)} e^{i\kappa_- (\tilde{x}P_+)t} \langle P_1, P_2 | O^a(\kappa_-; \tilde{x})_{\mu^2} | 0 \rangle |_{\tilde{x}P_- = \eta \tilde{x}P_+}. \quad (3.24)$$

It is interesting that a similar distribution function was already introduced by Chase [17] in the two photon production of two jets. In the limit  $\eta \rightarrow 1$  we obtain the known result for the production of a scalar meson by two photons for large  $Q^2$  [4,23].

Note that also here we generalize the meson wave function to a two-meson wave function in a straightforward way. Of course we cannot predict the dependence of this wave function on its internal parameters (this includes also its  $P_1 P_2$  dependence). This is in contrast to the large momentum transfer calculation where  $P_1 P_2 \rightarrow \infty$ . Here one introduces for each meson its own wave function and then it is clear that the  $P_1 P_2$  dependence of the process is included into the hard scattering part.

### C. Parton interpretation

Here we try to interpret the foregoing results.

There are two remarkable differences of the foregoing processes in comparison with the usually discussed forward or fixed angle scattering processes:

1. There appear generalized distribution amplitudes describing both hadrons simultaneously. These distribution functions depend, besides other variables, also on the scaling variable  $\eta$  defined above.
2. The leading contribution of the hard scattering part consists of an interaction of both photons with one and the same internal quark line. The momenta of the other quarks are not forced to be large, so that they are confined and can be described implicitly by the generalized distribution amplitude. Fig. 1 illustrates this remark for the case of the virtual Compton scattering. A standard treatment of exclusive pion (or nucleon) Compton scattering at fixed angles would include much more diagrams.

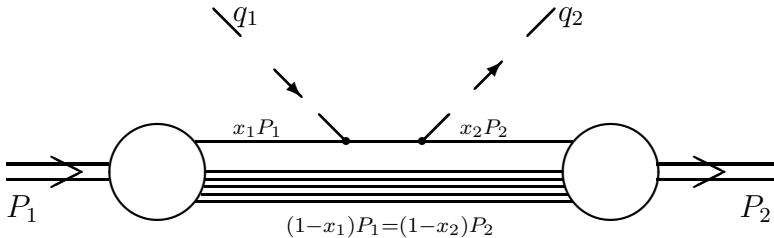


FIG. 1. Leading contribution for the virtual Compton scattering in the Bjorken region.

It remains an interpretation of  $t$  in the parton language as momentum fractions of the external hadrons. In the case of the exclusive virtual Compton scattering we obtain

$$t = \frac{x_1 P_1 + x_2 P_2}{P_1 + P_2} \quad (3.25)$$

in an infinite momentum frame with respect to  $P_1 + P_2$ . This variable is a natural generalization of the earlier introduced distribution parameters  $z$  or  $x$  of the parton distribution function and the meson wave function, respectively. We underline that the standard calculation of the pion Compton effect at fixed angles for example uses diagrams of Fig. 2.

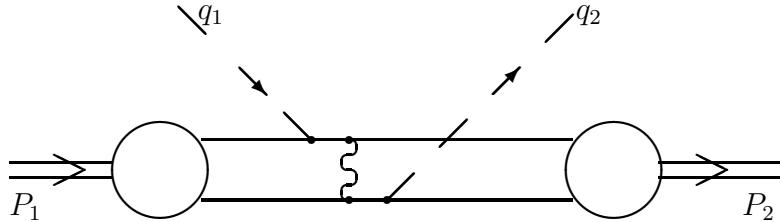


FIG. 2. Leading contribution for the virtual Compton amplitude at fixed angles.

Here in opposite to the foregoing case the photons act on both internal lines. Because of the large momentum transfer between both hadrons it is possible to introduce separate wave functions for each hadron. So the  $P_1 P_2$  dependence is a natural part of the hard scattering subprocess. If in addition we can go over to the Bjorken region, then both formalisms should be applicable and should lead to the same result.

#### IV. DERIVATION OF THE GENERAL EVOLUTION EQUATION

In this section, we derive an evolution equation for the distribution amplitude (2.10) which contains as limiting cases the Brodsky-Lepage and the Altarelli-Parisi equations. This evolution equation exploits the renormalization group equation of the light-ray operators

(2.5). For this reason we have to know the renormalization properties of these operators and especially their anomalous dimensions. Afterwards we can apply these results to expectation values of these operators which are contained in the distribution amplitude.

It is now well-known that bilocal operators where the distance between both positions of the involved field operators is light-like possess additional divergences. Therefore they need a special renormalization procedure. In the scalar case the renormalization of light-ray operators has been studied from a general point of view by S. A. Anikin and O. I. Zavialov [12]. These operators need special Z-factors depending on the parameters of these operators and additional anomalous dimensions [24]. A general thorough proof for the renormalization of these operators in QCD with covariant gauge fixing we do not know. However practical calculations of the Z-factors respectively the corresponding anomalous dimensions exist at one loop [4] and two loops for restricted variable ranges [9,10] for covariant gauge fixing as well as in the light-cone gauge.

It turns out that for a straightforward mathematical treatment the set of operators (2.5) is not well suited. We use, therefore, the more general operators (2.3)

$$O^a(\kappa_+, \kappa_-; \tilde{n}) = : \bar{\psi}(\kappa_1 \tilde{n})(\tilde{n} \gamma) \lambda^a U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n}) \psi(\kappa_2 \tilde{n}) :, \quad \kappa_{\pm} = (\kappa_2 \pm \kappa_1)/2. \quad (4.1)$$

These operators differ from the original ones by a translation. Their renormalization group equation reads formally

$$\begin{aligned} \mu \frac{d}{d\mu} O^a(\kappa_+, \kappa_-; \tilde{n})_{(\mu^2)} = & \int d\kappa'_+ d\kappa'_- [\gamma(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-; \alpha_s(\mu^2)) - 2\gamma_\psi(\alpha_s(\mu^2)) \\ & \times \delta(\kappa_+ - \kappa'_+) \delta(\kappa_+ - \kappa'_-)] O^a(\kappa'_+, \kappa'_-; \tilde{n})_{(\mu^2)}. \end{aligned} \quad (4.2)$$

This integral equation contains two integration variables. Using symmetry properties a more simple form of this equation will be derived in Appendix B [16], namely

$$\begin{aligned} \mu \frac{d}{d\mu} O^a(\kappa_+, \kappa_-; \tilde{n})_{(\mu^2)} = & \int dw_+ dw_- [\gamma(w_+, w_-; \alpha_s(\mu^2)) - 2\gamma_\psi(\alpha_s(\mu^2)) \\ & \times \delta(w_+) \delta(1 - w_-)] O^a(w_+ \kappa_- + \kappa_+, w_- \kappa_-; \tilde{n})_{(\mu^2)}. \end{aligned} \quad (4.3)$$

In the Appendix B we also investigate the anomalous dimensions  $\gamma(w_+, w_-)$  of the above defined operators. As result, we obtain the following support restriction (B25)

$$\gamma(w_+, w_-) \neq 0 \quad \text{for} \quad |w_{\pm}| \leq 1, |w_+ \pm w_-| \leq 1. \quad (4.4)$$

Additionally, from the transformation properties of the operators under charge conjugation it follows  $\gamma(w_+, w_-) = \gamma(-w_+, w_-)$  [Eq. (B32)].

Technically we investigate in Appendix B the divergent part of all one-particle-irreducible (1PI) diagrams with the insertion of one light-ray operator for scalar field theory and QCD in light-cone gauge. As essential tool we use the  $\alpha$ -representation. However we do not prove or carry out the renormalization procedure of the considered operator. For the investigation of the support properties of the Z-factors and the anomalous dimensions of these operators this is not necessary. So we study the support properties of the divergent term with respect to the parameters  $\kappa_+$  and  $\kappa_-$  of the inserted operator and the parameters  $\kappa'_+$  and  $\kappa'_-$  of

the appearing counter terms. This allows us to determine the support properties of the anomalous dimension of the light-ray operators.

After these preliminaries we turn to the derivation of the evolution equation. For this purpose we differentiate the general distribution amplitude  $q^a(t, \tau, \mu^2)$  with respect to the renormalization parameter  $\mu$ . Thereby we take into account its representation (2.10) in terms of matrix elements of the light-ray operator. The differentiation of this operator can be performed with the help of its renormalization group equation (4.3). A straightforward calculation runs as follows:

$$\begin{aligned} \mu \frac{d}{d\mu} q^a(t, \tau, \mu^2) &= \int \frac{d\kappa_- |\tilde{n}P_+|}{2\pi(\tilde{n}P_+)} e^{i\kappa_- (\tilde{n}P_+)t} \mu \frac{d}{d\mu} \langle P_2 | O^a(\kappa_+ = 0, \kappa_-; \tilde{n}) | P_1 \rangle |_{\tilde{n}P_- = \tau(\tilde{n}P_+)} \\ &= \int dw_- dw_+ \int \frac{d\kappa_- |\tilde{n}P_+|}{2\pi(\tilde{n}P_+)} e^{i\kappa_- (\tilde{n}P_+)t} [\gamma(w_+, w_-) - 2\gamma_\psi \\ &\quad \times \delta(w_+) \delta(1 - w_-)] \langle P_2 | O^a(\kappa_- w_+, \kappa_- w_-; \tilde{n}) | P_1 \rangle |_{\tilde{n}P_- = \tau(\tilde{n}P_+)}. \end{aligned} \quad (4.5)$$

As next step we exploit the translation invariance of the matrix element

$$\langle P_2 | O^a(\kappa_- w_+, \kappa_- w_-; \tilde{n}) | P_1 \rangle = \langle P_2 | O^a(\kappa_+ = 0, \kappa_- w_-; \tilde{n}) | P_1 \rangle e^{i\kappa_- w_+ (\tilde{n}P_-)} \quad (4.6)$$

and introduce the new variables  $\kappa'_- = \kappa_- w_-$  and  $t' = (t + \tau w_+)/w_-$  into Eq. (4.5):

$$\begin{aligned} \mu \frac{d}{d\mu} q^a(t, \tau, \mu^2) &= \int \frac{d^2 \underline{w}}{|w_-|} \int \frac{d\kappa'_- |\tilde{n}P_+|}{2\pi(\tilde{n}P_+)} e^{i(t + \tau w_+) \kappa'_- (\tilde{n}P_+)/w_-} [\gamma(w_+, w_-) - 2\gamma_\psi \\ &\quad \times \delta(w_+) \delta(1 - w_-)] \langle P_2 | O^a(\kappa'_+ = 0, \kappa'_-; \tilde{n}) | P_1 \rangle |_{\tilde{n}P_- = \tau(\tilde{n}P_+)} \\ &= \int dt' \int \frac{d^2 \underline{w}}{|w_-|} \delta\left(t' - \frac{t + \tau w_+}{w_-}\right) [\gamma(w_+, w_-) - 2\gamma_\psi \delta(w_+) \delta(1 - w_-)] \\ &\quad \times q^a(t', \tau, \mu^2). \end{aligned} \quad (4.7)$$

As result, we obtain the evolution equation

$$Q^2 \frac{d}{dQ^2} q^a(t, \tau, Q^2) = \int_{-1}^1 \frac{dt'}{|2\tau|} \left( \gamma\left(\frac{t}{\tau}, \frac{t'}{\tau}\right) - 2\gamma_\psi \delta\left(\frac{t}{\tau} - \frac{t'}{\tau}\right) \right) q^a(t', \tau, Q^2) \quad (4.8)$$

with the evolution kernel

$$\gamma(t, t') = \int dw_- \gamma(w_+ = t' w_- - t, w_-). \quad (4.9)$$

From the renormalization group invariance of  $\int dt q^a(t, \tau, \mu^2)$  (see Eq. (2.15)) it follows

$$\int dt \gamma(t, t') = 2\gamma_\psi, \quad (4.10)$$

so that the standard "+"-definition for the generalized function

$$[\gamma(t, t')]_+ = \gamma(t, t') - \delta(t - t') \int dt'' \gamma(t'', t') = \gamma(t, t') - 2\gamma_\psi \delta(t - t') \quad (4.11)$$

arises in a very natural way. We see that instead of the anomalous dimension there appears an evolution kernel which contains the anomalous dimension as an essential input. As it becomes clear later on it seems to be natural to speak of our general evolution kernel as of an extended BL-kernel. The reason for this is the following: Restricting  $\gamma(t, t')$  to the parameter region  $|t|, |t'| \leq 1$ , it coincides with the evolution kernel for the hadron wave function, but Eq. (4.8) provides us with a meaningfull definition also outside of this region.

In this way, we obtain evolution equations for all forward and nonforward matrix elements. To our knowledge, in the literature up to now only two examples have been investigated: the case of forward scattering and the case of the meson wave function. Here, both cases are contained as limits  $P_2 \rightarrow P_1$  or  $P_2 \rightarrow 0$  (see section II). We write these limits down in a condensed form:

- Evolution equation for the quark distribution function (forward scattering):

$$q^a(z, Q^2) = \lim_{\tau \rightarrow 0} q^a(z, \tau, Q^2),$$

$$Q^2 \frac{d}{dQ^2} q^a(z, Q^2) = \int_{-1}^1 \frac{dz'}{|z'|} P\left(\frac{z}{z'}; \alpha_s(Q^2)\right) q^a(z', Q^2), \quad (4.12)$$

$$|z'|^{-1} P\left(\frac{z}{z'}\right) = \lim_{\tau \rightarrow 0} \frac{1}{|2\tau|} \left[ \gamma\left(\frac{z}{\tau}, \frac{z'}{\tau}\right) \right]_+, \quad \text{Altarelli-Parisi-(Lipatov) kernel.}$$

$$(4.13)$$

- Evolution equation for hadron wave functions:

This case can be obtained as a formal limit too [compare the remarks after Eq. (2.12)].

$$\Phi^a(x = (1+t)/2, Q^2) = \lim_{\tau \rightarrow -1} q^a(t, \tau, Q^2),$$

$$Q^2 \frac{d}{dQ^2} \Phi^a(x, Q^2) = \int_0^1 dy V_{BL}(x, y; \alpha_s(Q^2)) \Phi^a(y, Q^2), \quad (4.14)$$

$$V_{BL}(x, y) = [\gamma(2x-1, 2y-1)]_{+}|_{0 \leq x, y \leq 1}, \quad \text{Brodsky-Lepage kernel.} \quad (4.15)$$

Here, the variables  $x = (1+t)/2$ ,  $y = (1+t')/2$  are restricted to  $0 \leq x, y \leq 1$ .

Note that both kernels (4.13) and (4.15) are calculated separately in QCD. From the above considerations it is obvious, however, that they have a common origin, namely the anomalous dimension  $\gamma(w_+, w_-)$  of the light-ray operators being hidden within the corresponding amplitudes. We return to this point in section V.

Let us add a remark concerning the distribution amplitude introduced for the nonforward virtual Compton scattering. This distribution amplitude appearing in (3.14) or (3.23) satisfies in the flavour nonsinglet case the evolution equation

$$Q^2 \frac{d}{dQ^2} q^a(t, \eta, Q^2) = \int_{-1}^1 \frac{dt'}{|2\eta|} \left[ \gamma\left(\frac{t}{\eta}, \frac{t'}{\eta}\right) \right]_+ q^a(t', \eta, Q^2). \quad (4.16)$$

Here, the distribution function depends on variables with a clear physical interpretation. It coincides formally with the definition (4.8) so that no further proof is needed. Nevertheless, this equation can be proved directly: It is possible to derive an evolution equation for  $f^a(z_+, z_-, Q^2)$  starting from the original renormalization group equation, and to use Eq. (2.17) for the amplitude  $q^a(t', \eta, Q^2)$ . This chain of relations is independent of the concrete physical meaning of the variables, so that the evolution equation (4.8) is valid for the changed variables too. In the singlet case we have a mixing problem with a gluonic contribution. This problem could be solved in a straightforward manner, too.

## V. THE EXTENDED BL-KERNEL, RELATIONS BETWEEN THE BL- AND AP-KERNELS

In section IV we derived the general evolution kernel (an extended BL-kernel) for distribution amplitudes containing the (restricted) BL-kernel and the AP-kernel as special cases. In QCD the BL-kernel is calculated as an evolution kernel for a restricted range of the variables. The interesting questions are: Exists there a possibility to determine the extended BL-kernel knowing this restricted kernel only? Furthermore, if such a procedure exists, is it then possible to apply it to the existing two-loop QCD calculations? If yes, then one should be able to perform the limit to the Altarelli-Parisi kernel which should then coincide with the already existing results obtained by straightforward computation. We solve these problems in the following three subsections.

### A. General Aspects

Here, we discuss the general solution of the problems stated above. Starting from the support properties of the anomalous dimension  $\gamma(w_+, w_-)$  and from the definition of the evolution kernel (4.9), we study its domain of definition. Afterwards, we study the extension procedure. It turns out that the restricted BL-kernel contains already the essential information and that an explicit continuation procedure can be prescribed.

The first question is: What is the correct region in the  $(t, t')$ -plane where the kernel  $\gamma(t, t')$  is defined in fact. To answer it, we start from the general representation (4.9) of this kernel

$$\gamma(t, t') = \int \int dw_- dw_+ \delta(w_+ - t + t' w_-) \gamma(w_+, w_-)$$

and use all known symmetry properties and support restrictions:

$$\gamma(w_+, w_-) = \gamma(-w_+, w_-), \quad |w_\pm| \leq 1, \quad |w_+ \pm w_-| \leq 1.$$

After some algebra we obtain the following representation in the  $(t, t')$ -plane [7]

$$\begin{aligned} \gamma(t, t') = & [\theta(t - t')\theta(1 - t) - \theta(t' - t)\theta(t - 1)]f(t, t') \\ & + [\theta(t' - t)\theta(1 + t) - \theta(t - t')\theta(-t - 1)]f(-t, -t') \\ & + [\theta(-t - t')\theta(1 + t) - \theta(t' + t)\theta(-t - 1)]g(-t, t') \\ & + [\theta(t + t')\theta(1 - t) - \theta(-t' - t)\theta(t - 1)]g(t, -t'), \end{aligned} \quad (5.1)$$

where the functions  $f(t, t')$  and  $g(t, t')$  are given by

$$\begin{aligned} f(t, t') &= \int_0^{\frac{1-t}{1-t'}} dw_- \gamma(w_+ = t - w_- t', w_-), \\ g(-t, t') &= \int_0^{\frac{1+t}{1-t'}} dw_- \gamma(w_+ = -t - w_- t', -w_-). \end{aligned} \quad (5.2)$$

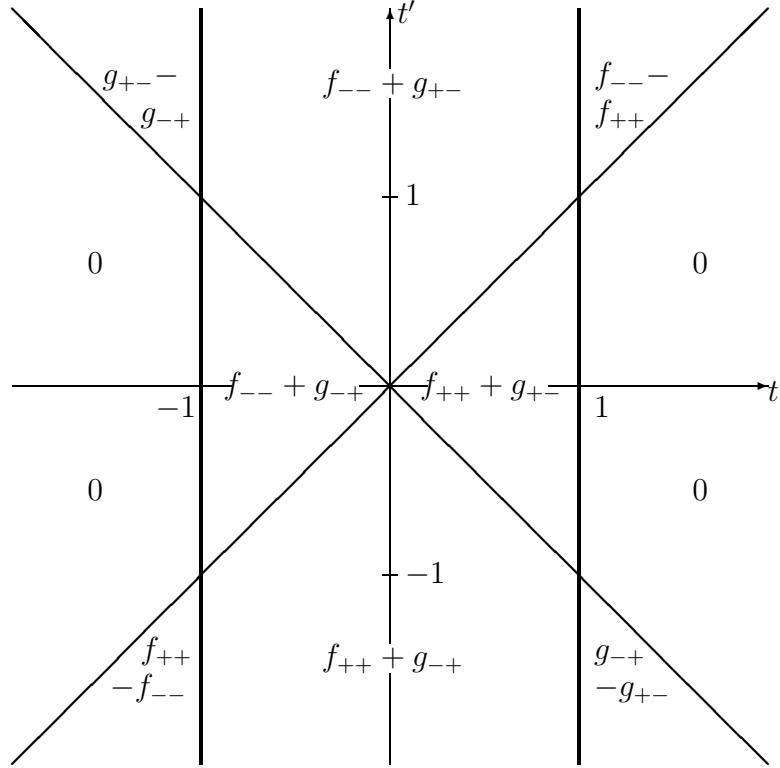


FIG. 3. Support of  $\gamma(t, t')$ , where  $f_{\pm\pm} = f(\pm t, \pm t')$ , and  $g_{\pm\mp} = g(\pm t, \mp t')$  are defined by Eqs. (5.2).

Note, that for  $|t|, |t'| > 1$  because of support restrictions the lower boundary in the integral representations (5.2) is not attained. However, in these regions only the differences

$$f(t, t') - f(-t, -t') \quad \text{and} \quad g(-t, t') - g(t, -t')$$

appear in Eq. (5.1), so that such undetermined contributions eliminate each other. It is clear, therefore, that the evolution kernel is defined in the complete  $(t, t')$ -plane as shown in Fig. 3.

The second question is now: Assuming we are able to calculate the anomalous dimension in the region  $-1 \leq t, t' \leq 1$ , is it possible to determine it in the whole  $(t, t')$ -plane? This problem contains the principal question whether the restricted BL-kernel contains already the full information on the general anomalous dimension. For solving this problem we perform a Fourier transform of the extended BL-kernel and later on of the restricted BL-kernel. It will be shown that because of the holomorphic properties of these functions it is possible to identify them.

So we perform a Fourier transform of the extended BL-kernel. Thereby, we take into account the known support restrictions explicitly

$$\begin{aligned}\tilde{\gamma}(\lambda, t') &= \int dt e^{i\lambda t} [\gamma(t, t')]_+ \\ &= \iint_{\substack{|w_+|, |w_-| \leq 1 \\ |w_+ \pm w_-| \leq 1}} dw_+ dw_- \gamma(w_+, w_-) (e^{i\lambda(w_+ + t' w_-)} - e^{i\lambda t'}) .\end{aligned}\quad (5.3)$$

Because of the finite integration region in the Fourier integral with respect to the variables  $w_+$ ,  $w_-$  the resulting Fourier transform of the anomalous dimension is an entire function of the variables  $\lambda$  and  $t'$  [25].

Now, we define a second entire function of the same variables by the Fourier transform of the restricted BL-kernel

$$\begin{aligned}\tilde{V}(\lambda, t') &= \int_{-1}^1 dt e^{i\lambda t} [V(t, t')]_+ \\ &= \iint_{\substack{|w_+|, |w_-| \leq 1 \\ |w_+ \pm w_-| \leq 1}} dw_+ dw_- \theta(1 - w_- t' - w_+) \theta(w_- t' + w_+ + 1) \\ &\quad \times \gamma(w_+, w_-) (e^{i\lambda(w_+ + t' w_-)} - e^{i\lambda t'}) .\end{aligned}\quad (5.4)$$

Both functions have to be compared. The essential point is that, due to the known support properties, for  $|t'| \leq 1$  it follows automatically also  $|t| \leq 1$  so that for  $|t'| \leq 1$  there exists a region where both expression  $\tilde{\gamma}(\lambda, t')$  and  $\tilde{V}(\lambda, t')$  are identical. Therefore, knowing the Fourier transform of the BL-kernel we can obtain the Fourier transform of the extended BL-kernel by analytic continuation. After performing the reversed transformation

$$[\gamma(t, t')]_+ = \int \frac{d\lambda}{2\pi} e^{-i\lambda t} \{\tilde{V}(\lambda, t')\}_{AC} \quad (5.5)$$

we obtain  $\gamma(t, t')$  completely.  $\{\tilde{V}(\lambda, t')\}_{AC}$  denotes formally the analytic continuation procedure to be carried out.

Consequently, it is sufficient to perform all diagrammatic calculations in the restricted region  $|t|, |t'| \leq 1$ . The essential information on the anomalous dimension is already contained inside of this region.

## B. The Extended BL-Kernel of QCD in Two-Loop Approximation

As an exercise to the foregoing investigations we will extend the Brodsky-Lepage kernel of QCD into the complete  $(t, t')$ -plane starting from the explicit one- and two-loop calculations of the restricted BL-kernel [9,10].

The known calculations of the evolution kernel [9,10] are performed in the range  $-1 \leq t, t' \leq 1$ , which was sufficient for studying the evolution of the hadron wave function. The difficult two-loop calculation have been done in the variables  $x = (1 + t)/2$ ,  $y = (1 + t')/2$ . So for the rest of the paper we shall prefer these variables too. The result of the above mentioned calculations is

$$V(x, y) = \frac{\alpha_s}{2\pi} c_F [V_0(x, y)]_+ + \left(\frac{\alpha_s}{2\pi}\right)^2 c_F [c_F V_F(x, y) + (c_G/2) V_G(x, y) + (N_F/2) V_N(x, y)]_+ + O(\alpha_s^3), \quad (5.6)$$

with

$$V_0(x, y) = \theta(y - x) F(x, y) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}, \quad (5.7a)$$

$$V_N(x, y) = \theta(y - x) v_N(x, y) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}, \quad (5.7b)$$

$$V_G(x, y) = \theta(y - x) v_G(x, y) + G(x, y) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}, \quad (5.7c)$$

$$V_F(x, y) = \theta(y - x) v_F(x, y) - \frac{x}{2\bar{y}} k(x) - G(x, y) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}. \quad (5.7d)$$

Here, the following auxiliary functions are used:

$$F(x, y) = \frac{x}{y} \left(1 - \frac{1}{x - y}\right), \quad (5.8a)$$

$$v_N(x, y) = -\frac{10}{9}F - \frac{2}{3}\frac{x}{y} - \frac{2}{3}F \ln\left(\frac{x}{y}\right), \quad (5.8b)$$

$$v_G(x, y) = \frac{67}{9}F + \frac{17}{3}\frac{x}{y} + \frac{11}{3}F \ln\left(\frac{x}{y}\right), \quad (5.8c)$$

$$\begin{aligned} v_F(x, y) &= -\frac{\pi^2}{3}F + \frac{x}{y} - \left(\frac{3}{2}F - \frac{x}{2\bar{y}}\right) \ln\left(\frac{x}{y}\right) - (F - \bar{F}) \ln\left(\frac{x}{y}\right) \\ &\quad \times \ln\left(1 - \frac{x}{y}\right) + \left(F + \frac{x}{2\bar{y}}\right) \ln^2\left(\frac{x}{y}\right) \\ &= -\frac{\pi^2}{3}F + \frac{x}{y} - \frac{3}{2}F \ln\left(\frac{x}{y}\right) - \left(F - \frac{1}{x - y}\right) \ln\left(\frac{x}{y}\right) \\ &\quad \times \ln\left(1 - \frac{x}{y}\right) + F \ln^2\left(\frac{x}{y}\right) + \frac{x}{2\bar{y}}k\left(\frac{x}{y}\right), \end{aligned} \quad (5.8d)$$

$$k(x) = \ln(x)(1 + \ln(x) - 2\ln(\bar{x})), \quad (5.8e)$$

$$\begin{aligned}
G(x, y) = & 2\theta(y - x)[\overline{F}\text{Li}_2(\overline{x}) + \overline{F}\ln(y)\ln(\overline{x}) - F\text{Li}_2(\overline{y})] + \theta(x + y - 1) \left[ 2(F - \overline{F}) \right. \\
& \left. \times \text{Li}_2\left(1 - \frac{x}{y}\right) + (F - \overline{F})\ln^2(y) - 2F\ln(y)\ln(x) + 2F\text{Li}_2(\overline{y}) - 2F\text{Li}_2(x) \right], \tag{5.8f}
\end{aligned}$$

where the Spence function  $\text{Li}_2$  is defined by

$$\text{Li}_2(x) = -\int_0^x dz \frac{\ln(1-z)}{z}. \tag{5.9}$$

As shorthand notation we have introduced

$$\overline{x} = 1 - x, \quad \overline{y} = 1 - y, \quad \overline{F} \equiv \overline{F}(x, y) = F(1 - x, 1 - y). \tag{5.10}$$

The group theoretical constants are  $c_F = 4/3$ ,  $c_G = 3$ .  $N_F$  denotes the number of quark flavors. Contrary to the usual notation of the kernel in Ref. [9,10] we have changed the notation by

$$G(x, y) + 2\theta(y - x)\overline{F}\ln(y)\ln(\overline{x}) \rightarrow G(x, y).$$

Furthermore, for convenience we have introduced the function  $k(x)$ , and in the last row of (5.8d) we have used the decomposition  $\overline{F}(x, y) = -\frac{x}{y} + \frac{1}{x-y}$ .

The obtained kernel  $V(x, y)$  is not defined for  $x = y$ , however the "+"-definition provides a regularization prescription.

The kernel (5.6) has to be extended using the foregoing results. Let us begin with the results of the one-loop calculations (5.7a) and (5.8a). According to our procedure, we have to perform an analytical continuation of the Fourier transform

$$\tilde{V}_0(\lambda, y) = 2 \int_0^1 dx \left[ \theta(y - x)F(x, y) + \left\{ \begin{array}{l} x \rightarrow \overline{x} \\ y \rightarrow \overline{y} \end{array} \right\}_+ e^{i\lambda(2x-1)} \right], \tag{5.11}$$

which may be written as

$$\begin{aligned}
\tilde{V}_0(\lambda, y) = & 2 \int_0^y dx F(x, y) \left( e^{i\lambda(2x-1)} - e^{i\lambda(2y-1)} \right) \\
& + 2 \int_0^{\overline{y}} d\overline{x} F(\overline{x}, \overline{y}) \left( e^{i\lambda(1-2\overline{x})} - e^{i\lambda(1-2\overline{y})} \right). \tag{5.12}
\end{aligned}$$

Both integrals are well defined and represent an analytical function with respect to the variables  $\lambda$  and  $y$ . In the first integral we introduce the variable  $z = x/y$  and in the second the variable  $z = \overline{x}/\overline{y}$  so that

$$\tilde{V}_0(\lambda, y) = 2 \int_0^1 dz y F(zy, y) \left( e^{i\lambda(2zy-1)} - e^{i\lambda(2y-1)} \right) + \left\{ \begin{array}{l} y \rightarrow \overline{y} \\ \lambda \rightarrow -\lambda \end{array} \right\}. \tag{5.13}$$

Obviously,  $yF(zy, y) = y - 1/(1 - z)$  is an entire function with respect to  $y$ , and the integrand  $yF(zy, y) \left( e^{i\lambda(2zy-1)} - e^{i\lambda(2y-1)} \right)$  has no singularities for  $0 \leq z \leq 1$ . Then the analytical continuation of  $\tilde{V}_0(\lambda, y)$  is given by

$$\{\tilde{V}_0(\lambda, y)\}_{AC} = 2 \int_0^1 dz \{yF(zy, y)\}_{AC} (e^{i\lambda(2zy-1)} - e^{i\lambda(2y-1)}) + \left\{ \begin{array}{l} y \rightarrow \bar{y} \\ \lambda \rightarrow -\lambda \end{array} \right\}, \quad (5.14)$$

where  $\{yF(zy, y)\}_{AC} = y - 1/(1 - z)$  denotes the analytical continuation with respect to the variable  $y$ . Obviously the inverse transformation provides

$$\begin{aligned} V_0^{ext}(x, y) &= \int_0^1 dz \{yF(zy, y)\}_{AC} (\delta(x - 2zy) - \delta(x - y)) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\} \\ &= \theta(1 - z)\theta(z)\text{sign}(y)\{F(zy, y)\}_{AC}|_{z=\frac{x}{y}} \\ &\quad - \delta(x - y) \int_0^1 dz \{yF(zy, y)\}_{AC} + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\} \\ &= \left[ \theta(1 - z)\theta(z)\text{sign}(y)\{F(zy, y)\}_{AC}|_{z=\frac{x}{y}} \right]_+ + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}. \end{aligned} \quad (5.15)$$

So we obtain as extended BL-kernel in one-loop approximation

$$\begin{aligned} V_0^{ext}(x, y) &= \gamma_0(t = 2x - 1, t' = 2y - 1) \\ &= \theta\left(1 - \frac{x}{y}\right)\theta\left(\frac{x}{y}\right)\text{sign}(y)\frac{x}{y}\left(1 - \frac{1}{x - y}\right) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\}. \end{aligned} \quad (5.16)$$

Note that this result can be easily checked by a direct calculation of  $\gamma_0(t, t')$ .

The same procedure can be performed for the two-loop contributions to the BL-kernel. The calculations are much more complicated. For convenience, we consider each term separately.

We start with the contributions coming from the functions  $v_N$ ,  $v_G$  and  $v_F$ . They have - as in the foregoing case - the property that the functions  $yv_N(zy, y)$ ,  $yv_G(zy, y)$  and  $yv_F(zy, y)$  are entire analytical functions of the variable  $y$ . Therefore, the extension of the corresponding part of the BL-kernel can be performed in the same manner: The extension of these contributions is obtained by substituting

$$\theta(y - x) \rightarrow \theta\left(1 - \frac{x}{y}\right)\theta\left(\frac{x}{y}\right)\text{sign}(y) \quad (5.17)$$

and performing the analytical continuation of the functions  $v_N$ ,  $v_G$  and  $v_F$ , e.g.

$$v_F(x, y) = \{v_F(zy, y)\}_{AC}|_{z=\frac{x}{y}}. \quad (5.18)$$

More problems occur for the contributions  $\frac{x}{2y}k(\frac{x}{y})$  and  $\frac{x}{2y}k(x)$ . These functions have a pole for  $y = 1$ . However, the Fourier transformation of

$$K(x, y) = \theta(y - x)\frac{x}{2y}k\left(\frac{x}{y}\right) - \frac{x}{2y}k(x) + \left\{ \begin{array}{l} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{array} \right\} \quad (5.19)$$

can be written in the following manner

$$\tilde{K}(\lambda, y) = \int_0^1 dz \frac{z}{y} [y^2 k(z) (e^{i\lambda(2zy-1)} - e^{i\lambda(2y-1)}) - \{y = 1\}] + \left\{ \begin{array}{l} y \rightarrow \bar{y} \\ \lambda \rightarrow -\lambda \end{array} \right\}. \quad (5.20)$$

Obviously, the integrand in Eq. (5.20) is an entire function, and we can use the procedure described above.

The extension of  $G(x, y)$  is more complicated. If we write down its Fourier transform in the same manner as in Eq. (5.13) then the integrand seems to contain poles and cuts. These are caused by the  $\overline{F}$ -function and by the  $\ln$ - and  $\text{Li}_2$ -functions, respectively. However, with the help of identities for the  $\text{Li}_2$ -functions it is possible to find such a representation in the region  $0 \leq y \leq 1$  that the integrand is an analytic function (without poles and cuts) for  $0 \leq \text{Re } y$ . Similarly, it is possible to find a representation where the integrand is an analytical function for  $\text{Re } y \leq 1$ . With other words, the Fourier transform of  $G(x, y)$  is in fact an entire function, but, opposite to the cases treated above, there does not exist a representation for the integrand in terms of  $\text{Li}_2$ -functions which is well suited for all values of  $y$ . Therefore, formally we can obtain the extension by performing the substitution (5.17) (of course,  $\theta(y - \overline{x}) \rightarrow \theta(1 - \frac{\overline{x}}{y})\theta(\frac{\overline{x}}{y})\text{sign}(y)$ ) and the analytical continuation of the corresponding functions,

$$\begin{aligned} G^{ext}(x, y) = & 2\theta\left(1 - \frac{x}{y}\right)\theta\left(\frac{x}{y}\right)\text{sign}(y)\{\overline{F}\text{Li}_2(\overline{x}) + \overline{F}\ln(y)\ln(\overline{x}) - F\text{Li}_2(\overline{y})\}_{AC} \\ & + \theta\left(1 - \frac{\overline{x}}{y}\right)\theta\left(\frac{\overline{x}}{y}\right)\text{sign}(y)\left\{2(F - \overline{F})\text{Li}_2\left(1 - \frac{x}{y}\right) + (F - \overline{F})\ln^2(y) \right. \\ & \left. - 2F\ln(y)\ln(x) + 2F\text{Li}_2(\overline{y}) - 2F\text{Li}_2(x)\right\}_{AC} + \begin{Bmatrix} x \rightarrow \overline{x} \\ y \rightarrow \overline{y} \end{Bmatrix}. \end{aligned} \quad (5.21)$$

As result, we find for  $y < 0$  and  $y > 1$  terms which contain an imaginary part. However, these cancel each other exactly so that in the physically interesting regions the extension provides in fact a real function.

As final result the extended BL-kernel reads:

$$\begin{aligned} [\gamma(2x - 1, 2y - 1)]_+ = & V^{ext}(x, y) \\ = & \frac{\alpha_s}{2\pi}c_F[V_0^{ext}(x, y)]_+ + \left(\frac{\alpha_s}{2\pi}\right)^2 c_F[c_F V_F^{ext}(x, y) + (c_G/2)V_G^{ext}(x, y) \\ & + (N_F/2)V_N^{ext}(x, y)]_+ + O(\alpha_s^3), \end{aligned} \quad (5.22)$$

where the extension of  $V_0$ ,  $V_F$ ,  $V_G$  and  $V_N$  (5.7a-5.8f) follows from the discussed procedure which is given by (5.17), (5.18) and (5.21). So, up to the changed  $\theta$ -structures the analytic expressions are formally identical to the originally calculated ones.

### C. Consistency between the QCD Calculation of the BL- and AP-Kernels

An interesting application of our formula (4.13) would be a check of the mutual consistency of the very complicated two-loop calculations of the BL-kernel [9,10] and the AP-kernel [8]. As it has been shown the extended BL-kernel should contain the AP-kernel as limiting case. Therefore, independent calculations of both kernels are not necessary. Otherwise, if they exist, then they must be consistent with each other.

The existence of a connection between the AP- and the BL-kernels follows already from a very simple argument. Indeed, looking at the local anomalous dimensions it seems to be

obvious that the general anomalous dimension matrix  $V_{nm} = \gamma_{nm} - 2\gamma_\psi\delta_{nm}$  corresponding to the BL-kernel [26]

$$\int_0^1 dx x^n V_{BL}(x, y) = \sum_{m=0}^n V_{nm} y^m \quad (5.23)$$

contains the diagonal matrix elements corresponding to the AP-kernel,

$$\int_{-1}^1 dz z^n P(z) = V_{nn}, \quad P(z) = \frac{1}{2\pi i} \int_{-c-i\infty}^{-c+i\infty} dz z^{-n-1} V_{nn}. \quad (5.24)$$

If the local anomalous dimensions would determine the corresponding kernels then it should be possible to determine the AP-kernel from the given BL-kernel. Starting from this observation there were several incomplete trials to solve this problem [9].

Here, we shall determine the AP-kernel from the known extended BL-kernel. In the language of the local anomalous dimension we can say that in this way an independent check for the correctness of the diagonal anomalous dimension will be performed. A check for the nondiagonal elements of the anomalous dimension matrix has been carried out by exploiting consequently conformal symmetry breaking [11]. Therefore, the considerations given in the following enable us to conclude that the two-loop approximation of the BL-kernel has been correctly computed.

To obtain the AP-kernel practically, we have to apply Eq. (4.13) which we reproduce here for convenience

$$P(z) = \lim_{\tau \rightarrow 0} |\tau|^{-1} [\gamma(2x - 1, 2y - 1)]_+ |_{x=\frac{z}{\tau}, y=\frac{1}{\tau}} \quad (5.25)$$

Let us remark that according to this equation it is just the added new region ( $|t|, |t'| > 1$ ) which is essential for the determination of the AP-kernel.

For technical reasons, we first discuss the  $\theta$ -structure and then we consider the limiting procedure for the coefficient functions contained in Eqs. (5.8a-5.8f). The limits  $\tau \rightarrow 0$  with  $x = z/\tau$ ,  $y = 1/\tau$  for the  $\theta$ -functions are

$$\left. \begin{array}{l} \theta\left(\frac{x}{y}\right)\theta\left(1 - \frac{x}{y}\right) \\ \theta\left(\frac{1-x}{1-y}\right)\theta\left(1 - \frac{1-x}{1-y}\right) \end{array} \right\} \rightarrow \theta(z)\theta(1-z), \quad (5.26)$$

$$\left. \begin{array}{l} \theta\left(\frac{1-x}{y}\right)\theta\left(1 - \frac{1-x}{y}\right) \\ \theta\left(\frac{x}{1-y}\right)\theta\left(1 - \frac{x}{1-y}\right) \end{array} \right\} \rightarrow \theta(-z)\theta(1+z). \quad (5.27)$$

It is important for further calculations that "related  $\theta$ -structures", i.e. structures which turn into each other under  $x \leftrightarrow \bar{x} = 1 - x$  and  $y \leftrightarrow \bar{y} = 1 - y$  - have the same limit. We note, that all expressions have the following typical structures

$$S_1(x, y) = \theta\left(\frac{x}{y}\right)\theta\left(1 - \frac{x}{y}\right) \text{sign}(y)T_1(x, y) + \theta\left(\frac{\bar{x}}{\bar{y}}\right)\theta\left(1 - \frac{\bar{x}}{\bar{y}}\right) \text{sign}(\bar{y})T_1(\bar{x}, \bar{y}) \quad (5.28)$$

or

$$S_2(x, y) = \theta\left(\frac{\bar{x}}{y}\right) \theta\left(1 - \frac{\bar{x}}{y}\right) \text{sign}(y) T_2(x, y) + \theta\left(\frac{x}{\bar{y}}\right) \theta\left(1 - \frac{x}{\bar{y}}\right) \text{sign}(\bar{y}) T_2(\bar{x}, \bar{y}). \quad (5.29)$$

So we have to determine the following limits:

$$\lim_{\tau \rightarrow 0} |\tau|^{-1} S_1\left(\frac{z}{\tau}, \frac{1}{\tau}\right) = \theta(z) \theta(1-z) \lim_{\tau \rightarrow 0} \tau^{-1} \left\{ T_1\left(\frac{z}{\tau}, \frac{1}{\tau}\right) - T_1\left(1 - \frac{z}{\tau}, 1 - \frac{1}{\tau}\right) \right\} \quad (5.30)$$

and

$$\lim_{\tau \rightarrow 0} |\tau|^{-1} S_2\left(\frac{z}{\tau}, \frac{1}{\tau}\right) = \theta(-z) \theta(1+z) \lim_{\tau \rightarrow 0} \tau^{-1} \left\{ T_2\left(\frac{z}{\tau}, \frac{1}{\tau}\right) - T_2\left(1 - \frac{z}{\tau}, 1 - \frac{1}{\tau}\right) \right\}. \quad (5.31)$$

As short notation we introduce the symbol LIM,

$$\text{LIMIT}(x, y) = \lim_{\tau \rightarrow 0} \tau^{-1} \left\{ T\left(\frac{z}{\tau}, \frac{1}{\tau}\right) - T\left(1 - \frac{z}{\tau}, 1 - \frac{1}{\tau}\right) \right\}. \quad (5.32)$$

The determination of these limits will be performed in Appendix E.

Putting together all results obtained there we get an expression for the AP-kernel following from the extended BL-kernel (5.6-5.8f). In analogy with Eq. (5.6) we write

$$P(z) = \frac{\alpha_s}{2\pi} c_F [P_0(z)]_+ + \left(\frac{\alpha_s}{2\pi}\right)^2 c_F [c_F P_F(z) + (c_G/2) P_G(z) + (N_F/2) P_N(z)]_+ + O(\alpha_s^3). \quad (5.33)$$

Let us compute each term separately

$$\begin{aligned} P_0(z) &= \lim_{\tau \rightarrow 0} |\tau|^{-1} V_0\left(\frac{z}{\tau}, \frac{1}{\tau}\right) \\ &= \theta(z) \theta(1-z) \frac{1+z^2}{1-z} \end{aligned} \quad (5.34)$$

according to (E1b).

$$\begin{aligned} P_N(z) &= \lim_{\tau \rightarrow 0} |\tau|^{-1} V_N\left(\frac{z}{\tau}, \frac{1}{\tau}\right) \\ &= -\theta(z) \theta(1-z) \frac{2}{3} \left[ \frac{1+z^2}{1-z} \left( \ln(z) + \frac{5}{3} \right) + 2(1-z) \right] \end{aligned} \quad (5.35)$$

because of (E1a-E1c).

$$\begin{aligned} P_G(z) &= \lim_{\tau \rightarrow 0} |\tau|^{-1} V_G\left(\frac{z}{\tau}, \frac{1}{\tau}\right) \\ &= \theta(z) \theta(1-z) \left[ \frac{1+z^2}{1-z} \left( \ln^2(z) + \frac{11}{3} \ln(z) + \frac{67}{9} - \frac{\pi^2}{3} \right) \right] \end{aligned}$$

$$\begin{aligned}
& + 2(1+z)\ln(z) + \frac{40}{3}(1-z) \Big] \\
& + \theta(-z)\theta(1+z) \left[ 2 \frac{1+|z|^2}{1+|z|} \left( \text{Li}_2 \left( \frac{|z|}{1+|z|} \right) - \text{Li}_2 \left( \frac{1}{1+|z|} \right) + \frac{1}{2} \ln^2(|z|) \right. \right. \\
& \quad \left. \left. - \ln(|z|) \ln(1+|z|) \right) + 2(1+|z|)\ln(|z|) + 4(1-|z|) \right]. \tag{5.36}
\end{aligned}$$

Here we have taken into account (E1a-E1c,E1g,E1h). For the last term we obtain the result:

$$\begin{aligned}
P_F(z) &= \lim_{\tau \rightarrow 0} |\tau|^{-1} V_F \left( \frac{z}{\tau}, \frac{1}{\tau} \right) \\
&= \theta(z)\theta(1-z) \left[ -\frac{1+z^2}{1-z} \left( \ln^2(z) + \frac{3}{2} \ln(z) + 2 \ln(z) \ln(1-z) \right) \right. \\
&\quad \left. + \frac{1+3z^2}{2(1-z)} \ln^2(z) - \frac{1}{2} (3+7z) \ln(z) - 5(1-z) \right] \\
&\quad - \theta(-z)\theta(1+z) \left[ 2 \frac{1+|z|^2}{1+|z|} \left( \text{Li}_2 \left( \frac{|z|}{1+|z|} \right) - \text{Li}_2 \left( \frac{1}{1+|z|} \right) + \frac{1}{2} \ln^2(|z|) \right. \right. \\
&\quad \left. \left. - \ln(|z|) \ln(1+|z|) \right) + 2(1+|z|)\ln(|z|) + 4(1-|z|) \right]. \tag{5.37}
\end{aligned}$$

The obtained result for the AP-kernel (5.33-5.37) is completely equivalent to that of Ref. [8]. It also includes in a natural way the second order contributions stemming from the internal anti-quark lines.

Let us add a remark concerning the regularization prescription of the considered kernel. It is very important, that our regularization prescription for the extended BL-kernel turns after performing the limiting process, automatically into the well accepted regularization prescription for the AP-kernel. If we start with (4.11) and (5.25)

$$\lim_{\tau \rightarrow 0} |\tau|^{-1} \left[ \gamma \left( \frac{z}{\tau}, \frac{1}{\tau} \right) - \delta \left( \frac{z}{\tau} - \frac{1}{\tau} \right) \int_{-\infty}^{\infty} dx' \gamma \left( 2x' - 1, \frac{2}{\tau} - 1 \right) \right] \tag{5.38}$$

then the limit of the  $\theta$ -structure restricts the integration range in Eq. (5.38). The substitution  $x' = z'/\tau$  leads to

$$\lim_{\tau \rightarrow 0} |\tau|^{-1} \left[ \gamma \left( \frac{z}{\tau}, \frac{z}{\tau} \right) - \delta(z-1) \int_{-\infty}^{\infty} dz' \gamma \left( \frac{z'}{\tau}, \frac{1}{\tau} \right) \right] = P(z) - \delta(z-1) \int dz' P(z') \tag{5.39}$$

This completes our considerations.

## APPENDIX A: VIRTUAL COMPTON SCATTERING AMPLITUDE IN LEADING APPROXIMATION

The aim of this Appendix is to study the behaviour of the helicity amplitudes of the Compton scattering amplitude in the generalized Bjorken region. The helicity amplitudes are defined by

$$T(\lambda', \lambda) = \varepsilon_2^\mu(\lambda') T_{\mu\nu} \varepsilon_1^\nu(\lambda), \quad (\text{A1})$$

where  $\varepsilon_i(\lambda)$  denotes the polarization vector of the virtual photon ( $i = 1, 2$ ):

$$\begin{aligned} \varepsilon_i^\mu(\lambda) &= (0, \vec{\varepsilon}_i(\lambda)), \quad \vec{\varepsilon}_i(\lambda) \vec{q}_i = 0, \quad \vec{\varepsilon}_i(\lambda) \vec{\varepsilon}_i(\lambda') = \delta_{\lambda\lambda'}, \quad \text{for } \lambda, \lambda' = 1, 2, \\ \varepsilon_i^\mu(\lambda = 3) &= |q_i^2|^{-1/2} (|\vec{q}_i|, q_i^0 \vec{q}_i / |\vec{q}_i|), \\ &= |q_i^2|^{-1/2} (q_i^\mu - q_i^2 c_i^\mu / c_i q_i), \quad c_i^\mu = (1, -\vec{q}_i / |\vec{q}_i|), \\ \varepsilon_i^\mu(\lambda = 0) &= |q_i^2|^{-1/2} q_i^\mu. \end{aligned} \quad (\text{A2})$$

It will be shown that in leading order they can be expressed as

$$T(\lambda', \lambda) \approx \begin{cases} \frac{1}{2} \varepsilon_2^\mu(\lambda') \varepsilon_{1\mu}(\lambda) T_\nu^\nu & \text{for } \lambda', \lambda = 1, 2, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A3})$$

so that in section III we need to consider the more simple expression  $T_\nu^\nu$  only.

To get these results at first we remark that because of the current conservation  $q_2^\mu T_{\mu\nu} = 0$  and  $T_{\mu\nu} q_1^\nu = 0$  the helicity amplitudes  $T(\lambda', \lambda)$  vanish for  $\lambda' = 0$  or  $\lambda = 0$ . For the same reason a kinematical suppression of the longitudinal degrees of freedom occurs, for example

$$\varepsilon_2^\mu(\lambda = 3) T_{\mu\nu} = |q_2^2|^{-1/2} q_2^\mu T_{\mu\nu} \pm \frac{\sqrt{|q_2^2|}}{c_2 q_2} c_2^\mu T_{\mu\nu} = \pm \frac{\sqrt{|q_2^2|}}{c_2 q_2} c_2^\mu T_{\mu\nu} \quad \text{for } q_2^2 \gtrless 0 \quad (\text{A4})$$

is suppressed because the factor

$$\frac{\sqrt{|q_i^2|}}{c_i q_i} = \frac{\sqrt{|(q \pm P_-/2)^2|}}{c_i (q \pm P_-/2)} \approx \frac{\sqrt{Q^2}}{c_i q} = O\left(\frac{1}{\sqrt{Q^2}}\right) \quad i = 1, 2. \quad (\text{A5})$$

vanishes asymptotically.

For further simplification we need explicite representations and approximations of the scattering amplitude. Because of our kinematical assumptions, in  $x$ -space the scattering amplitude (3.1) is dominated by contributions arising from the neighbourhood of the light-cone. So it is possible to apply the light-cone expansion of the current product. Instead of the standard expansion in terms of local operators we use the more effective expansion based on light-ray operators [12,22]. This expansion relies on a perturbative expansion of the product of the electromagnetic currents, too. The crucial difference between both expansions is the following: Whereas usually the light-cone expansion follows from a suitable Taylor expansion leading to the "local" light-cone operators, here a corresponding Fourier transform generates the nonlocal ("light-ray") operators. In leading order we obtain [4]

$$\begin{aligned}
T J_\mu \left( \frac{x}{2} \right) J_\nu \left( -\frac{x}{2} \right) &\approx \int d\kappa_+ d\kappa_- F^a(x^2, \kappa_+, \kappa_-, \mu^2) \Gamma_{\mu\nu}^{\alpha\beta} \tilde{x}_\alpha O_\beta^a(\kappa_+ \tilde{x}, \kappa_- \tilde{x})_{(\mu^2)} \\
&+ \int d\kappa_+ d\kappa_- \hat{F}^a(x^2, \kappa_+, \kappa_-, \mu^2) \epsilon_{\mu\nu}^{\alpha\beta} \tilde{x}_\alpha \hat{O}_\beta^a(\kappa_+ \tilde{x}, \kappa_- \tilde{x})_{(\mu^2)},
\end{aligned} \tag{A6}$$

where  $\Gamma_{\mu\nu}^{\alpha\beta} = g_{\mu\nu} g^{\alpha\beta} - g_\mu^\alpha g_\nu^\beta - g_\mu^\beta g_\nu^\alpha$  and  $\epsilon_{\mu\nu}^{\alpha\beta}$  denotes the  $\epsilon$ -tensor and

$$\tilde{x}(x, \rho) = x + \rho \frac{x\rho}{\rho^2} \left( \sqrt{1 - \frac{x^2 \rho^2}{(x\rho)^2}} - 1 \right)$$

is a light-like vector determined by  $x$  and parameterized by a fixed constant vector  $\rho$ . In the tree approximation we obtain for the coefficient functions  $F^a(x^2, \kappa_+, \kappa_-, \mu^2)$  and  $\hat{F}^a(x^2, \kappa_+, \kappa_-, \mu^2)$ :

$$\begin{aligned}
F^a(x^2, \kappa_+, \kappa_-) &= \frac{ic^a}{2\pi^2(x^2 - i\epsilon)^2} \delta(\kappa_+) (\delta(\kappa_- - 1/2) - \delta(\kappa_- + 1/2)) \\
\hat{F}^a(x^2, \kappa_+, \kappa_-) &= \frac{ic^a}{2\pi^2(x^2 - i\epsilon)^2} \delta(\kappa_+) (\delta(\kappa_- - 1/2) + \delta(\kappa_- + 1/2)), \\
c_a &= \frac{2}{9} \delta_{a0} + \frac{1}{6} \delta_{a3} + \frac{1}{6\sqrt{3}} \delta_{a8} \quad \text{for flavour SU(3)}. \tag{A7}
\end{aligned}$$

In this approximation the gluon operators are automatically suppressed, and only the quark operators contribute:

$$O_\beta^a(\kappa_+ \tilde{x}, \kappa_- \tilde{x}) = : \bar{\psi}(\kappa_1 \tilde{x}) \gamma_\beta \lambda^a U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \psi(\kappa_2 \tilde{x}) :, \tag{A8}$$

$$\hat{O}_\beta^a(\kappa_+ \tilde{x}, \kappa_- \tilde{x}) = : \bar{\psi}(\kappa_1 \tilde{x}) i \gamma^5 \gamma_\beta \lambda^a U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \psi(\kappa_2 \tilde{x}) :, \quad \kappa_\pm = (\kappa_2 \pm \kappa_1)/2. \tag{A9}$$

Here,  $U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})$  is the standard phase factor (2.5), and  $\lambda^a$  determines the flavour content, additionally we defined  $\lambda^0 \equiv 1$  so that the summation over the index  $a$  includes also the flavour singlet case. The choice  $\mu^2 = Q^2$  for the renormalization point justifies the perturbative calculation of the coefficient functions.

The nonperturbative part of the Compton scattering amplitude is contained in the matrix elements of the quark operators. Leaving aside the nonessential  $\kappa_+$ -dependent exponential [in fact it drops out because of  $\delta(\kappa_+)$  in Eqs. (A7)]

$$\langle P_2 | O_\beta^a(\kappa_+ \tilde{x}, \kappa_- \tilde{x}) | P_1 \rangle = \langle P_2 | O_\beta^a(\kappa_- \tilde{x}) | P_1 \rangle e^{i\kappa_+ (\tilde{x} P_-)}$$

we write down the general structure of the matrix elements of these operators between scalar or pseudoscalar mesons or spin averaged baryon states

$$\begin{aligned}
\langle P_2 | T O_\beta^a(\kappa_- \tilde{x}) | P_1 \rangle &= P_{+\beta} t_1^a(\kappa_- \tilde{x}, P_i, \mu^2) + P_{-\beta} t_2^a(\kappa_- \tilde{x}, P_i, \mu^2) \\
&+ \kappa_- \tilde{x}_\beta M^2 t_{nl}^a(\kappa_- \tilde{x}, P_i, \mu^2), \tag{A10}
\end{aligned}$$

$$\langle P_2 | T \hat{O}_\beta^a(\kappa_- \tilde{x}) | P_1 \rangle = \epsilon_{\beta\gamma\delta\epsilon} P_+^\gamma P_-^\delta \kappa_- \tilde{x}^\epsilon \hat{t}^a(\kappa_- \tilde{x}, P_i, \mu^2). \tag{A11}$$

The mass factor  $M^2 = P_i^2$  in the third term of Eq. (A10) has to be introduced for dimensional reasons. This term does not contribute in leading order. Inserting Eqs. (A6), (A10), (A11) into Eq. (3.1) we are able to write formally

$$\begin{aligned}
T_{\mu\nu}(P_+, P_-, q) &= \Gamma_{\mu\nu\alpha\beta} \left( P_+^\beta \frac{\partial}{\partial q_\alpha} T_1(P_+, P_-, q) + P_-^\beta \frac{\partial}{\partial q_\alpha} T_2(P_+, P_-, q) \right) \\
&\quad - i \epsilon_{\mu\nu\alpha\beta} \epsilon_{\gamma\delta\epsilon}^\beta P_+^\gamma P_-^\delta \frac{\partial}{\partial q_\alpha} \frac{\partial}{\partial q_\epsilon} \hat{T}(P_+, P_-, q),
\end{aligned} \tag{A12}$$

where

$$T_i(P_+, P_-, q) = \int d^4x \int d\kappa_+ d\kappa_- e^{ixq + i\kappa_+(\tilde{x}P_-)} F^a(x^2, \kappa_+, \kappa_-) t_i^a(\kappa_-\tilde{x}, P_i, Q^2). \tag{A13}$$

A corresponding formula is valid for  $\hat{T}(P_+, P_-, q)$ . The r.h.s. of Eq. (A12) can be simplified further: Since

$$\begin{aligned}
\frac{\partial}{\partial q_\alpha} T_i(P_+, P_-, q) &= \left( 2q^\alpha \frac{\partial}{\partial q^2} + \left( \frac{\partial}{\partial q_\alpha} \xi \right) \frac{\partial}{\partial \xi} + \left( \frac{\partial}{\partial q_\alpha} \eta \right) \frac{\partial}{\partial \eta} \right) T_i(\xi, \eta, Q^2) \\
&= \left( -2q^\alpha \frac{\partial}{\partial Q^2} - \frac{2q^\alpha + \xi P_+^\alpha}{qP_+} \frac{\partial}{\partial \xi} + \frac{P_-^\alpha - \eta P_+^\alpha}{qP_+} \frac{\partial}{\partial \eta} \right) T_i(\xi, \eta, Q^2)
\end{aligned} \tag{A14}$$

we can estimate

$$\begin{aligned}
\frac{\partial}{\partial q_\alpha} T_i(P_+, P_-, q) &\approx -2q^\alpha \left( \frac{\partial}{\partial Q^2} + \frac{1}{qP_+} \frac{\partial}{\partial \xi} \right) T_i(\xi, \eta, Q^2) \\
&\approx -\frac{2q^\alpha}{Q^2} \left( Q^2 \frac{\partial}{\partial Q^2} + \xi \frac{\partial}{\partial \xi} \right) T_i(\xi, \eta, Q^2) \quad \text{for } Q^2, qP_+, qP_- \rightarrow \infty.
\end{aligned} \tag{A15}$$

Using

$$\varepsilon_i(\lambda)q = -\varepsilon_i(\lambda)(q_i + (-1)^i P_-) = -(-1)^i \varepsilon_i(\lambda)P_- \quad \text{for } \lambda = 1, 2, \tag{A16}$$

and the explicit expression for  $\Gamma_{\mu\nu\alpha\beta}$ , we obtain after some algebra the leading contributions for the transversal helicity amplitudes:

$$T(\lambda', \lambda) \approx \varepsilon_2^\mu(\lambda') \varepsilon_{1\mu}(\lambda) \left( P_+ \frac{\partial}{\partial q} T_1 + P_- \frac{\partial}{\partial q} T_2 \right). \tag{A17}$$

Here we took into consideration, that the  $\hat{T}$  amplitude gives asymptotically nonleading contribution, which follows from similar arguments. Using Eq. (A12), a direct calculation of the trace  $T_{\mu\nu}$  shows

$$T_\mu^\mu \approx 2 \left( P_+ \frac{\partial}{\partial q} T_1 + P_- \frac{\partial}{\partial q} T_2 \right). \tag{A18}$$

Comparing Eqs. (A17) and (A18), we obtain the desired result (A3).

## APPENDIX B: PROPERTIES OF ANOMALOUS DIMENSIONS OF LIGHT-RAY OPERATORS

This Appendix is devoted to a general discussion of the support properties of the anomalous dimensions of light-ray operators (2.3),

$$O^a(\kappa_+, \kappa_-; \tilde{n}) =: \bar{\psi}(\kappa_1 \tilde{n})(\tilde{n} \gamma) \lambda^a U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n}) \psi(\kappa_2 \tilde{n}) :, \quad \kappa_{\pm} = (\kappa_2 \pm \kappa_1)/2, \quad \tilde{n}^2 = 0. \quad (\text{B1})$$

These operators differ from the original ones (2.5) by a translation. Their renormalization group equation reads [16]

$$\mu \frac{d}{d\mu} O^a(\underline{\kappa}; \tilde{n})_{(\mu^2)} = \int d^2 \underline{\kappa}' \left( \gamma(\underline{\kappa}, \underline{\kappa}'; g(\mu^2)) - 2\gamma_\psi(g(\mu^2)) \delta^{(2)}(\underline{\kappa} - \underline{\kappa}') \right) O^a(\underline{\kappa}'; \tilde{n})_{(\mu^2)}. \quad (\text{B2})$$

For convenience we divide the anomalous dimension of the operator into the anomalous dimension of the 1PI vertex function (with two external momenta)  $\gamma(\underline{\kappa}, \underline{\kappa}') = \gamma(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-)$  and a part which is proportional to the anomalous dimension of the quark field  $\gamma_\psi$ . As a shorthand notation we use  $O^a(\underline{\kappa}; \tilde{n}) = O^a(\kappa_+, \kappa_-; \tilde{n})$ ,  $d^2 \underline{\kappa}' = d\kappa'_+ d\kappa'_-$  and  $\delta^{(2)}(\underline{\kappa} - \underline{\kappa}') = \delta(\kappa_+ - \kappa'_+) \delta(\kappa'_- - \kappa'_-)$ .

In the following, we investigate the anomalous dimensions  $\gamma(\underline{\kappa}, \underline{\kappa}')$  of the above defined operator using the  $\alpha$ -representation for 1-particle-irreducible (1PI) diagrams containing this operator insertion. As result, we shall obtain the support restriction (B25), i. e.  $|w_{\pm}| \leq 1$ ,  $|w_+ \pm w_-| \leq 1$  with  $w_+ = (\kappa'_+ - \kappa_+)/\kappa_-$ ,  $w_- = \kappa'_-/\kappa_-$  and the special variable dependence  $\gamma(\underline{\kappa}, \underline{\kappa}') = \kappa_-^{-2} \gamma(w_+, w_-)$ . Additionally, from the transformation properties of the operators (B30) under charge conjugation it follows  $\gamma(w_+, w_-) = \gamma(-w_+, w_-)$ . In all what follows we give the more technical proof of these results.

In principle, we have to investigate the renormalization of gauge invariant operators on the light-cone. Thereby we should have in mind:

1. It is essential that these nonlocal operators are defined on a light-ray. This changes their renormalization properties and induces a close correspondence to the local operators known from the standard light-cone expansion [27].
2. From the renormalization procedure of gauge invariant operators it is well-known that during the renormalization process there may appear gauge variant and ghost operators. In the complete renormalization matrix they appear in a triangular form, so that the anomalous dimensions of the physical operators remain unchanged. Moreover these operators have vanishing matrix elements between physical particle states. For this reason, we do not need to consider such contributions [28].
3. Up to these restrictions the operators (B1) form a complete operator basis of minimal twist and minimal dimension.

The anomalous dimension can be determined from the 1PI vertex function of the operator to be studied [22,24]. For instance, by using dimensional regularization it can be shown that in the minimal subtraction scheme the following simple rule is valid [29]

$$\gamma(\underline{\kappa}, \underline{\kappa}'; g) = g \frac{\partial}{\partial g} Z^{[1]}(\underline{\kappa}, \underline{\kappa}'; g) \quad (B3)$$

where  $Z^{[1]}$  can be obtained from the residue of the 1PI vertex function  $R'G(p_1, p_2; g; \epsilon|O^a(\underline{\kappa}))$  with respect to the parameter  $\epsilon = (4 - n)/2$ . In detail, by  $R'G(p_1, p_2; g; \epsilon|O^a(\underline{\kappa}))$  we denote the 1PI vertex function (with two external momenta  $p_1$  and  $p_2$ ) containing the operator insertion where all subdiagrams are renormalized, and only the overall renormalization is not carried out. The  $\epsilon$ -expansion of this function reads [30]

$$R'G(p_1, p_2; \epsilon|O^a(\underline{\kappa}; \tilde{n})) = G^{[0]}(p_1, p_2|O^a(\underline{\kappa}; \tilde{n})) + \frac{1}{\epsilon} G^{[1]}(p_1, p_2|O^a(\underline{\kappa}; \tilde{n})) + \dots \quad (B4)$$

In general, the  $1/\epsilon$  term is given as an integral with respect to the  $\underline{\kappa}$ -variables of the  $Z$ -factor with the vertex

$$O_V^a(\underline{\kappa}; \tilde{n}; \underline{p}) = (\tilde{n}\gamma)\lambda^a e^{i\underline{\kappa}_+(\tilde{n}p_+) + i\underline{\kappa}_-(\tilde{n}p_-)} \quad (B5)$$

of the bare operator (B1). We have, therefore,

$$G^{[1]}(p_1, p_2|O^a(\underline{\kappa}; \tilde{n})) = \int d^2 \underline{\kappa}' Z^{[1]}(\underline{\kappa}, \underline{\kappa}') O_V^a(\underline{\kappa}'; \tilde{n}; \underline{p}). \quad (B6)$$

Using the special form (B5) of the operator vertex we can write

$$(\tilde{n}\gamma)\lambda^a Z^{[1]}(\underline{\kappa}, \underline{\kappa}') = \int \frac{d(\tilde{n}p_+)}{2\pi} \int \frac{d(\tilde{n}p_-)}{2\pi} e^{-i\underline{\kappa}'_+(\tilde{n}p_+) - i\underline{\kappa}'_-(\tilde{n}p_-)} G^{[1]}(p_1, p_2|O^a(\underline{\kappa}; \tilde{n})). \quad (B7)$$

Because of Eqs. (B3) and (B4) the investigation of the support properties of the anomalous dimension requires the knowledge of the support properties of all possible diagrams contributing to  $R'G(p_1, p_2; g; \epsilon|O^a(\underline{\kappa}))$ .

The standard method for the investigation of Green's functions is the application of the  $\alpha$ -representation. We use the following formula for the propagator

$$\frac{P(k)}{m^2 - k^2 - i\rho} = \lim_{\xi \rightarrow 0} P\left(\frac{1}{2i} \frac{\partial}{\partial \xi}\right) i \int_0^\infty d\alpha e^{i\alpha(k^2 - m^2 + i\rho) + i2\xi k}, \quad (B8)$$

where  $P(k)$  denotes a well specified polynomial in the momentum  $k$ . Also a complicated matrix structure of the vertices can be included into this polynomial. With these remarks the  $\alpha$ -representation for a 1PI diagram with  $L$  internal lines and  $V$  vertices and two external legs can be written as [22]

$$G_\Gamma(p_1, p_2) = \int_0^\infty d\underline{\alpha} \prod_{l=1}^L P_l\left(\frac{1}{2i} \frac{\partial}{\partial \xi_l}\right) e^{-i\alpha_l(m_l - i\rho)} f_\Gamma(\underline{\alpha}; \underline{p}; \underline{\xi})|_{\underline{\xi}=0}, \quad (B9)$$

where

$$f_\Gamma(\underline{\alpha}; \underline{p}; \underline{\xi}) = \frac{\text{const.}}{D^{n/2}(\underline{\alpha})} e^{i \left\{ \sum_{i,j=1}^2 p_i A_{ij}(\underline{\alpha}) p_j + 2 \sum_{l=1}^L \sum_{i=1}^2 \xi_l B_{li}(\underline{\alpha}) p_i - \sum_{k,l=1}^L \xi_k K_{kl}(\underline{\alpha}) \xi_l \right\}}. \quad (B10)$$

Thereby, the regularization of the ultra-violet divergencies is ensured by dimensional regularization, i.e. by an analytical continuation from  $n = 4$  to  $n = 4 - 2\epsilon$ ,  $\epsilon > 0$ . For later convenience, we have displayed the polynomial structure with respect to variables  $p_i$  and  $\xi_l$  explicitly and introduced coefficients depending on  $\alpha$ -parameters only. Thus, our notation differs slightly from that of Ref. [22]; furthermore, in contrast to Ref. [22] we have absorbed the overall factor  $1/D$  of the exponent into the definition of  $A_{ij}$ ,  $B_{li}$  and  $K_{kl}$ . Anyway, the coefficients  $A_{ij}$ ,  $B_{li}$  and  $K_{kl}$  may be simply deduced from the general rules given there.

For our purpose we need diagrams containing one insertion of the light-cone operator (B1). Of course, the vertex corresponding to this operator is complicated. However, if we consider QCD in axial gauge  $\tilde{n}^\mu A_\mu = 0$ , then the phase factor  $U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n})$  does not contribute, and only the bare vertex (B5) remains. Moreover, ghost operators do not exist. The drawback of this procedure is the comparatively complicated structure of the gluon propagator [31]

$$D_{ab}^{\mu\nu} = \delta_{ab} \frac{-i}{k^2 + i\rho} \left[ g^{\mu\nu} - \frac{\tilde{n}^\mu k^\nu + \tilde{n}^\nu k^\mu}{2} \left( \frac{1}{\tilde{n}k + i\tau} + \frac{1}{\tilde{n}k - i\tau} \right) \right]. \quad (\text{B11})$$

The  $\tilde{n}$ -dependent terms contained in the second part of this propagator need additional considerations. For simplicity we choose for the regularization of the unphysical pole term  $(\tilde{n}k)^{-1}$  the principal value prescription. It is well-known that this procedure generates non-local counter terms. This problem can be avoided by using the Mandelstam or Leibbrandt prescription [32]. However, in our case this is not necessary, because the considered physical quantities are independent of the gauge fixing procedure and therefore, do not depend on the prescription of the unphysical poles. As usual, the spurious divergences are subtracted by hand (see also the two-loop calculation in Ref. [9]). Below we shall show that the contributions of the unphysical poles do not change the support property of the anomalous dimension. So we shall ignore them; in fact this means that we study the scalar theory first.

The operator vertex (B5) may be included into each diagram as a special first vertex connected through the lines 1 and 2 to the remaining part of the diagram. In the  $\alpha$ -representation, we have

$$G_\Gamma(\underline{p}|O(\underline{\kappa}; \tilde{n})) = O_V\left(\underline{\kappa}; \tilde{n}; \frac{1}{2i} \frac{\partial}{\partial \underline{\xi}}\right) G_\Gamma(\underline{p}, \underline{\xi})|_{\underline{\xi}=0}, \quad (\text{B12})$$

with

$$O_V(\underline{\kappa}; \tilde{n}; \underline{p}) = e^{i\kappa_+(\tilde{n}p_+) + i\kappa_-(\tilde{n}p_-)}.$$

The differential operator in the exponential shifts the variables according to  $\xi_1 \rightarrow \xi_1 + \frac{\kappa_1}{2}\tilde{n}$  and  $\xi_2 \rightarrow \xi_2 + \frac{\kappa_2}{2}\tilde{n}$ . This shift can be taken into account explicitly, so that the  $\alpha$ -representation with the special operator insertion takes the form

$$\begin{aligned} G_\Gamma(\underline{p}|O(\underline{\kappa}; \tilde{n})) &= \int_0^\infty d\underline{\alpha} \prod_{l=1}^L P_l \left( \frac{1}{2i} \frac{\partial}{\partial \xi_l} \right) e^{-i\alpha_l(m_l - i\rho)} f_\Gamma(\underline{\alpha}; \underline{p}; \underline{\xi}) \\ &\times \exp i \left\{ \sum_{i,j=1}^2 \kappa_i B_{ij}(\tilde{n}p_j) - \sum_{i=1}^2 \sum_{l=1}^L \kappa_i K_{il}(\tilde{n}\xi_l) \right\} |_{\underline{\xi}=0}. \end{aligned} \quad (\text{B13})$$

Here we have used  $\tilde{n}^2 = 0$  and the symmetry relation  $K_{kl}(\underline{\alpha}) = K_{lk}(\underline{\alpha})$  which is proved in Appendix C. After summation over all allowed graphs we obtain the 1PI vertex function

$$G(\underline{p}|O(\underline{\kappa}; \tilde{n})) = \sum_{\Gamma} G_{\Gamma}(\underline{p}|O(\underline{\kappa}; \tilde{n})). \quad (\text{B14})$$

Let us now turn to the support properties of the anomalous dimension with respect to the variables  $\underline{\kappa}$  and  $\underline{\kappa}'$ . We use the fact that the partly unrenormalized 1PI vertex function  $G(\underline{p}|O(\underline{\kappa}; \tilde{n}))$ , the renormalized functions  $RG(\underline{p}|O(\underline{\kappa}; \tilde{n}))$  and the anomalous dimension  $\gamma(\underline{\kappa}, \underline{\kappa}')$  have the same support. So we study the support of the vertex function (B14). Its support restrictions follow from the properties of the  $\kappa$ -dependent part at  $\xi_1 = \dots = \xi_L = 0$  in the exponential:

$$\begin{aligned} \sum_{i,j=1}^2 \kappa_i B_{ij}(\tilde{n}p_j) &= \\ &= \frac{\kappa_+}{2} [(B_{11} + B_{12} + B_{21} + B_{22})(\tilde{n}p_+) - (B_{11} - B_{12} + B_{21} - B_{22})(\tilde{n}p_-)] \\ &\quad - \frac{\kappa_-}{2} [(B_{11} + B_{12} - B_{21} - B_{22})(\tilde{n}p_+) - (B_{11} - B_{12} - B_{21} + B_{22})(\tilde{n}p_-)]. \end{aligned} \quad (\text{B15})$$

In Appendix C we prove

$$B_{11} + B_{21} = 1, \quad B_{12} + B_{22} = 1, \quad 0 \leq B_{ij} \leq 1, \quad K_{1k} + K_{2k} = 0, \quad K_{kl} = K_{lk}, \quad (\text{B16})$$

which suggests the definitions

$$B_+ = B_{11} - B_{22}, \quad B_- = 1 - B_{11} - B_{22}, \quad |B_+| \leq 1, \quad |B_-| \leq 1, \quad |B_+ \pm B_-| \leq 1. \quad (\text{B17})$$

By taking into account these relations we obtain

$$\begin{aligned} \sum_{i,j=1}^2 \kappa_i B_{ij}(\tilde{n}p_j) - \sum_{i=1}^2 \sum_{l=1}^L \kappa_i K_{il}(\tilde{n}\xi_l) &= \kappa_+(\tilde{n}p_+) - \kappa_-[B_+(\tilde{n}p_+) + B_-(\tilde{n}p_-)] \\ &\quad + \sum_{l=1}^L \kappa_- K_{1l}(\tilde{n}\xi_l) \end{aligned} \quad (\text{B18})$$

so that the expression for the 1PI vertex function takes the form

$$\begin{aligned} G(\underline{p}|O(\underline{\kappa}; \tilde{n})) &= \sum_{\Gamma} \int_0^{\infty} d\underline{\alpha} \prod_{l=1}^L P_l \left( \frac{1}{2i} \frac{\partial}{\partial \xi_l} \right) e^{-i\alpha_l(m_l - i\rho)} f_{\Gamma}(\underline{\alpha}; \underline{p}; \underline{\xi}) \\ &\quad \times \exp i \left\{ \kappa_+(\tilde{n}p_+) - \kappa_-[B_+(\tilde{n}p_+) + B_-(\tilde{n}p_-)] + \sum_{l=1}^L \kappa_- K_{1l}(\tilde{n}\xi_l) \right\} |_{\underline{\xi}=0}. \end{aligned} \quad (\text{B19})$$

The differentiation with respect to  $\xi$  generates the scalar products  $p_i p_j$ ,  $\kappa_-(\tilde{n}p_+)$  as well as  $\kappa_-(\tilde{n}p_-)$  so that we are able to write

$$G(\underline{p}|O(\underline{\kappa}; \tilde{n})) = \sum_{\Gamma} \int_0^{\infty} d\underline{\alpha} g_{\Gamma}(\underline{\alpha}; p_i p_j; \kappa_- \tilde{n} \underline{p}) e^{i\kappa_+(\tilde{n}p_+) - i\kappa_- [B_+(\tilde{n}p_+) + iB_-(\tilde{n}p_-)]}, \quad (\text{B20})$$

where

$$\begin{aligned} g_{\Gamma}(\underline{\alpha}; p_i p_j; \kappa_- \tilde{n} \underline{p}) &= \prod_{l=1}^L P_l \left( \frac{1}{2i} \frac{\partial}{\partial \xi_l} \right) e^{-i\alpha_l(m_l - i\rho)} f_{\Gamma}(\underline{\alpha}; \underline{p}; \underline{\xi}) \\ &\times e^{i\{\kappa_+(\tilde{n}p_+) - \kappa_- [B_+(\tilde{n}p_+) + B_-(\tilde{n}p_-)] + \sum_{l=1}^L \kappa_- K_{1l}(\tilde{n}\xi_l)\}}|_{\underline{\xi}=0}. \end{aligned} \quad (\text{B21})$$

According to the definition of the anomalous dimension (B3), (B4) and (B7), we perform the Fourier transform and find

$$\begin{aligned} \gamma(\underline{\kappa}, \underline{\kappa}') &= g \frac{\partial}{\partial g} \text{res} \int \frac{d(\tilde{n}p_+)}{2\pi} \int \frac{d(\tilde{n}p_-)}{2\pi} R' \sum_{\Gamma} \int_0^{\infty} d\underline{\alpha} g_{\Gamma}(\underline{\alpha}; p_i p_j; \kappa_- \tilde{n} \underline{p}) \\ &\times \exp i\{\kappa_+(\tilde{n}p_+) - \kappa_- [B_+(\tilde{n}p_+) + B_-(\tilde{n}p_-)] - \kappa'_+(\tilde{n}p_+) - \kappa'_-(\tilde{n}p_-)\}, \end{aligned} \quad (\text{B22})$$

where the  $R'$ -operation acts onto the coefficient function  $g_{\Gamma}$  only. For convenience, we introduce the variables [16]

$$w_+ = \frac{\kappa'_+ - \kappa_+}{\kappa_-}, \quad w_- = \frac{\kappa'_-}{\kappa_-} \quad (\text{B23})$$

and receive

$$\gamma(\underline{\kappa}, \underline{\kappa}') = \kappa_-^{-2} g \frac{\partial}{\partial g} \text{res} R' \sum_{\Gamma} \int_0^{\infty} d\underline{\alpha} g_{\Gamma} \left( \underline{\alpha}; p_i p_j; i \frac{\partial}{\partial w_+}, i \frac{\partial}{\partial w_-} \right) \delta(B_+ + w_+) \delta(B_- + w_-). \quad (\text{B24})$$

Because of the restrictions (B17) of the variables  $B_+$  and  $B_-$  we conclude

$$\gamma(w_+, w_-) \neq 0 \quad \text{for} \quad |w_{\pm}| \leq 1, |w_+ \pm w_-| \leq 1 \quad (\text{B25})$$

$$\gamma(w_+, w_-) = \kappa_-^2 \gamma(\underline{\kappa}, \underline{\kappa}'). \quad (\text{B26})$$

Note that the introduction of  $\gamma(w_+, w_-)$  was suggested by the special variable dependence of Eq. (B24).

Let us turn back now to QCD in axial gauge. As already mentioned, we have to apply a very complicated gluon propagator. It is in the spirit of the foregoing considerations to use the following  $\alpha$ -representation

$$\begin{aligned} D_{ab}^{\mu\nu} &= -\delta_{ab} \int_0^{\infty} d\alpha \int_0^{\infty} d\beta \left[ g^{\mu\nu} + \frac{i}{2} \left( \tilde{n}^{\mu} \frac{\partial}{\partial 2i\xi_{\nu}} + \tilde{n}^{\nu} \frac{\partial}{\partial 2i\xi_{\mu}} \right) \right. \\ &\times \left. \left( e^{i\{\beta \tilde{n} \frac{\partial}{\partial 2i\xi} + i\tau\}} - e^{-i\{\beta \tilde{n} \frac{\partial}{\partial 2i\xi} - i\tau\}} \right) \right] e^{i\alpha(k^2 + i\rho) + i2\xi k} |_{\xi=0}. \end{aligned} \quad (\text{B27})$$

If we insert this propagator instead of the usual one into the  $\alpha$ -representation of a diagram, then all investigations can be performed in a similar manner as before. However, it is more instructive to use instead of the  $(\alpha, \beta)$ -representation (B27) the formal expression

$$D_{ab}^{\mu\nu} = -\delta_{ab} \int_0^\infty d\alpha \left[ g^{\mu\nu} - \frac{1}{2} \left( \tilde{n}^\mu \frac{\partial}{\partial 2i\xi_\nu} + \tilde{n}^\nu \frac{\partial}{\partial 2i\xi_\mu} \right) \left( \frac{1}{\tilde{n} \frac{\partial}{\partial 2i\xi} + i\tau} + \frac{1}{\tilde{n} \frac{\partial}{\partial 2i\xi} - i\tau} \right) \right] \times e^{i\alpha(k^2 + i\rho) + i2\xi k}. \quad (\text{B28})$$

By writing now the  $\alpha$ -representation of a graph, we have to include also the pre-factors of the gluon propagators

$$g^{\mu\nu} - \frac{1}{2} \left( \tilde{n}^\mu \frac{\partial}{\partial 2i\xi_\nu} + \tilde{n}^\nu \frac{\partial}{\partial 2i\xi_\mu} \right) \left( \frac{1}{\tilde{n} \frac{\partial}{\partial 2i\xi} + i\tau} + \frac{1}{\tilde{n} \frac{\partial}{\partial 2i\xi} - i\tau} \right) \quad (\text{B29})$$

into the original product of the polynomials, so that instead of a polynomial there appears finally a rational function in the  $\xi$ -derivative. This may introduce new singularities. However, the expression in the exponential which is responsible for the support properties of the anomalous dimensions remains unchanged. This establishes the general support restrictions (B25) for QCD too.

Of course, for special operators there exist additional restrictions. The operator (B1) transforms under charge conjugation  $C$  in the following way (see Appendix D)

$$C(O^a(\kappa_+, \kappa_-; \tilde{n})) C^+ = \mp O^a(\kappa_+, -\kappa_-; \tilde{n}). \quad (\text{B30})$$

If we apply the charge conjugation to the renormalization group equation (B2) then we get

$$\mu \frac{d}{d\mu} O^a(\kappa_+, -\kappa_-; \tilde{n}) = \int d^2 \underline{\kappa}' \left( \gamma(\underline{\kappa}, \underline{\kappa}') - 2\gamma_\psi \delta^{(2)}(\underline{\kappa} - \underline{\kappa}') \right) O^a(\kappa'_+, -\kappa'_-; \tilde{n}). \quad (\text{B31})$$

The substitution  $\underline{\kappa} \rightarrow -\underline{\kappa}$ ,  $\underline{\kappa}' \rightarrow -\underline{\kappa}'$  and the comparison with the definition of the operator (B1) and Eq. (B2) leads to  $\gamma(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-) = \gamma(\kappa_+, -\kappa_-, \kappa'_+, -\kappa'_-)$ , which is equivalent to

$$\gamma(w_+, w_-) = \gamma(-w_+, w_-). \quad (\text{B32})$$

## APPENDIX C: PROPERTIES OF THE COEFFICIENTS OF THE $\alpha$ -REPRESENTATION

In this Appendix we prove the relations (B16). We study the coefficients  $D(\underline{\alpha})$ ,  $B_{li}(\underline{\alpha})$  and  $K_{kl}(\underline{\alpha})$  of the  $\alpha$ -representation of 1PI-Feynman graphs with an operator insertion. We assume that the graph contains  $V$  vertices (the vertex corresponding to the operator insertion is the first vertex),  $N$  external and  $L$  internal lines. To each internal line there corresponds one  $\alpha$  parameter. According to the momentum flow we define an orientation of the diagram. Instead of the coefficients  $B_{li}$  and  $K_{kl}$  appearing in the  $\alpha$ -representation we investigate the coefficients

$$b_{li}(\underline{\alpha}) = B_{li}(\underline{\alpha}) D(\underline{\alpha}), \quad k_{kl}(\underline{\alpha}) = K_{kl}(\underline{\alpha}) D(\underline{\alpha}), \quad (\text{C1})$$

which are homogeneous polynomials with respect to all  $\alpha$  and linear functions of each  $\alpha$  [22]. They are given by topological formulas.  $D$  is the so-called chord set product sum

$$D(\underline{\alpha}) = \sum_{T \in \{T\}} \left( \prod_{l \notin T} \alpha_l \right). \quad (\text{C2})$$

Here,  $\{T\}$  denotes a set of trees. A tree of the 1PI-graph is generated by cutting  $(L - V)$  lines of the graph so that a connected graph remains which does not contain loops. The degree of homogeneity of  $D$  is  $L - V$ . We note further definitions. If we consider a special line  $l$ , then a tree containing this line is denoted by  $T_l$ . If we cut this line, then we divide the considered tree into two half-trees corresponding to two simple connected diagrams. The half-tree lying in the direction of the orientation of the line  $l$  is denoted by  $T_l^+$ .

After these explanations we write down the definition of the coefficients  $b_{li}(\underline{\alpha})$  with the degree of homogeneity  $(L - V)$

$$b_{li}(\underline{\alpha}) = \sum_{\substack{T_l \in \{T\} \\ i \in T_l^+}} \left( \prod_{k \notin T} \alpha_k \right). \quad (\text{C3})$$

Here, the sum runs over all trees containing the line  $l$  whereas the considered half-tree has to contain the external vertex  $i$ . The set of these trees is denoted by  $\{T_l\}_{|i \in T_l^+}$ . Since  $\{T_l\}_{|i \in T_l^+} \subset \{T\}$  and  $\alpha_k \geq 0$  for each  $k$ , it follows from the definition of  $D(\underline{\alpha})$  and  $b_{li}(\underline{\alpha})$  that

$$0 \leq B_{li}(\underline{\alpha}) = \frac{b_{li}(\underline{\alpha})}{D(\underline{\alpha})} \leq 1, \quad (\text{C4})$$

which is the first relation (B16).

To prove the identity

$$B_{1i}(\underline{\alpha}) + B_{2i}(\underline{\alpha}) = \frac{b_{1i}(\underline{\alpha}) + b_{2i}(\underline{\alpha})}{D(\underline{\alpha})} = 1, \quad i = 1, \dots, V, \quad (\text{C5})$$

we have to show that  $\{T\}$  is just the disjoint union of the sets  $\{T_1\}_{|i \in T_1^+}$  and  $\{T_2\}_{|i \in T_2^+}$ . Since  $T \in \{T\}$  is connected, it has to contain the line  $l_1$  or  $l_2$  (see Fig. 4). Therefore  $T$  is contained in the sets  $\{T_1\}_{|i \in T_1^+}$  or  $\{T_2\}_{|i \in T_2^+}$ . Moreover  $\{T\} = \{T_1\}_{|i \in T_1^+} \cup \{T_2\}_{|i \in T_2^+}$ . It remains to show that the sets  $\{T_1\}_{|i \in T_1^+}$  and  $\{T_2\}_{|i \in T_2^+}$  are disjoint. If the vertex  $i$  is contained in a tree  $T$  belonging to the sets  $\{T_1\}_{|i \in T_1^+}$  and  $\{T_2\}_{|i \in T_2^+}$  then it is possible to find a path from the vertex  $i$  over the line  $l_1$  to the operator vertex and over the line  $l_2$  back to the vertex  $i$ . But this is not possible because  $T$  cannot contain a loop. Therefore the set  $\{T\}$  is the disjoint union of the sets  $\{T_1\}_{|i \in T_1^+}$  and  $\{T_2\}_{|i \in T_2^+}$  which proves Eq. (C5).

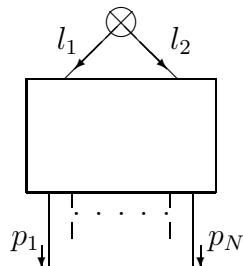


FIG. 4. Topology of the 1PI-Feynman graphs for the light-cone operators in light-cone gauge. The box symbolizes the connected  $(N + 2)$ -point function. The lines  $l_1$  and  $l_2$  are contained in a loop  $c$ .

For the investigation of the properties of the  $K_{kl}(\underline{\alpha})$  we start from the definition

$$k(\underline{\alpha}, \underline{\xi}) = \sum_{k,l=1}^L \xi_k k_{kl}(\underline{\alpha}) \xi_l = \sum_{T_c \in \{T_c\}} \left( \prod_{l \notin T_c} \alpha_l \right) \left( \sum_{j \in c} (\pm \xi_j) \right)^2. \quad (\text{C6})$$

Here,  $T_c$  denotes a pseudo tree (this means a tree to which we add a line in such a way that the new structure contains a loop  $c$ ). The sum  $(\sum_{j \in c} (\pm \xi_j))^2$  runs over all lines  $j$  which form the loop  $c$ . When the line  $j \in c$  points along the orientation of the loop  $c$  then we have to write a plus sign, in the opposite case a minus sign. Obviously the differentiation with respect to  $\xi$  provides the symmetry relation  $K_{kl}(\underline{\alpha}) = K_{lk}(\underline{\alpha})$ . Furthermore, the  $\xi_1$ - and  $\xi_2$ -dependent part of relation (C6) is determined by those  $T_c$  in which  $l_1$  and  $l_2$  are contained in the loop  $c$ . Then, because of our conventions, the lines  $l_1$  and  $l_2$  have different orientations so that we can write

$$k(\underline{\alpha}, \underline{\xi}) = k(\underline{\alpha}, \xi_1 - \xi_2, \xi_3, \dots, \xi_L). \quad (\text{C7})$$

The desired relation  $K_{1l}(\underline{\alpha}) = -K_{2l}(\underline{\alpha})$  is then a simple consequence of Eq. (C7).

## APPENDIX D: CHARGE SYMMETRY

We prove the following behaviour of the renormalized operators  $O^a(\kappa_+, \kappa_-)$  under charge ( $C$ )-conjugation:

$$CO^a(\kappa_+, \kappa_-) e^{iS_I} C^+ = \mp O^a(\kappa_+, -\kappa_-). \quad (\text{D1})$$

Let us begin with the C-conjugation of the elementary fields.

$$\begin{aligned} C\psi_\alpha^i(x)C^+ &= \sum_\beta c_{\alpha\beta} \bar{\psi}_\beta^i(x), & CA^\mu(x)C^+ &= -A^\mu(x), \\ C\bar{\psi}_\alpha^i(x)C^+ &= \sum_\beta \psi_\beta^i(x) \overset{T-1}{c}_{\beta\alpha}, \end{aligned} \quad (\text{D2})$$

where  $i$  and  $\alpha, \beta$  denote the flavour and the spinor indices of the quark field  $\psi$ . The transposed matrix of  $c := \{c_{\alpha\beta}\}$  is denoted by  $\overset{T}{c}$ . Using the relation

$$\overset{T-1}{c} \gamma_\mu c = \overset{T}{\gamma}_\mu \quad (\text{D3})$$

we find for the charge conjugation of the operator  $:\bar{\psi}(x)\gamma_\mu \lambda^a U(x, y)\psi(y):$

$$\begin{aligned} C : \bar{\psi}(x)\gamma_\mu \lambda^a U(x, y)\psi(y) : C^+ &= \sum_{\alpha\beta} : \psi_\alpha^i(x) (\overset{T-1}{c} \gamma_\mu c)_{\alpha\beta} \lambda_{ij}^a [CU(x, y)C^+] \bar{\psi}_\beta^i(y) : \\ &= \sum_{\alpha\beta} : \psi_\alpha^i(x) (\gamma_\mu)_{\beta\alpha} \lambda_{ij}^a [CU(x, y)C^+] \bar{\psi}_\beta^i : \\ &= - : \bar{\psi}(y)\gamma_\mu \lambda^a U(y, x)\psi(x) :. \end{aligned} \quad (\text{D4})$$

where we have used the anticommutation rule of the fermionic field as well as

$$CU(x, y)C^+ = P \exp \left\{ -ig \int_y^x dz_\mu CA^\mu(z)C^+ \right\} = U(y, x). \quad (\text{D5})$$

The hermitean generators  $\lambda^a$  satisfy the relation  $\lambda^a = \pm \lambda^a$ . Consequently, setting in Eq. (D4)  $x = \kappa_1 \tilde{n}$ ,  $y = \kappa_2 \tilde{n}$ , we find

$$C : \bar{\psi}(\kappa_1 \tilde{n}) \gamma_\mu \lambda^a U(\kappa_1 \tilde{n}, \kappa_2 \tilde{n}) \psi(\kappa_2 \tilde{n}) : C^+ = \mp : \bar{\psi}(\kappa_2 \tilde{n}) \gamma_\mu \lambda^a U(\kappa_2 \tilde{n}, \kappa_1 \tilde{n}) \psi(\kappa_1 \tilde{n}) : \quad (\text{D6})$$

or

$$CO^a(\kappa_+, \kappa_-)C^+ = \mp O^a(\kappa_+, -\kappa_-), \quad \kappa_\pm = (\kappa_2 \pm \kappa_1)/2. \quad (\text{D7})$$

In QCD  $C$ -invariance leads to relation (D1).

## APPENDIX E: LIMITS

After some algebra we obtain the following results for the structures appearing in the BL-kernels:

$$\text{LIM} \frac{x}{y} = 1 - z, \quad (\text{E1a})$$

$$\text{LIM} F(x, y) = \frac{1 + z^2}{1 - z}, \quad (\text{E1b})$$

$$\text{LIM} F(x, y) \ln \left( \frac{x}{y} \right) = \frac{1 + z^2}{1 - z} \ln(z) + 1 - z, \quad (\text{E1c})$$

$$\text{LIM} \frac{x}{2y} \ln \left( \frac{x}{y} \right) = -\frac{1}{2} ((1 + z) \ln(z) + 1 - z), \quad (\text{E1d})$$

$$\text{LIM} \left( F(x, y) + \frac{x}{2y} \right) \ln^2 \left( \frac{x}{y} \right) = \frac{1 + 3z^2}{2(1 - z)} \ln^2(z) + (1 - z) \ln(z), \quad (\text{E1e})$$

$$\text{LIM} [F(x, y) - \bar{F}(x, y)] \ln \left( \frac{x}{y} \right) \ln \left( 1 - \frac{x}{y} \right) = 2 \frac{1 + z^2}{1 - z} \ln(z) \ln(1 - z), \quad (\text{E1f})$$

$$\begin{aligned} & \text{LIM} [\bar{F}(x, y) \ln(y) \ln(\bar{x}) - F(x, y) \text{Li}_2(\bar{y}) + \bar{F}(x, y) \text{Li}_2(\bar{x})] \\ &= -\frac{1 + z^2}{1 - z} \left( \frac{\pi^2}{6} - \frac{1}{2} \ln^2(z) \right) + (1 + z) \ln(z) + 2(1 - z), \end{aligned} \quad (\text{E1g})$$

$$\begin{aligned}
\text{LIM}F(x, y) & \left[ 2\text{Li}_2\left(1 - \frac{x}{y}\right) + \ln^2(y) - 2\ln(x)\ln(y) + 2\text{Li}_2(\bar{y}) - 2\text{Li}_2(x) + \ln^2(\bar{y}) \right. \\
& \quad \left. + 2\text{Li}_2\left(1 - \frac{\bar{x}}{\bar{y}}\right) \right] \\
& = 2\frac{1+|z|^2}{1+|z|} \left( \text{Li}_2\left(\frac{|z|}{1+|z|}\right) - \text{Li}_2\left(\frac{1}{1+|z|}\right) + \frac{1}{2}\ln^2(|z|) - \ln(|z|)\ln(1+|z|) \right) \\
& \quad + 2(1+|z|)\ln(|z|) + 4(1-|z|), \tag{E1h}
\end{aligned}$$

$$\text{LIM} \left[ \frac{x}{2(\bar{y})} \ln(x)(1 + \ln(x) - 2\ln(\bar{x})) + \left\{ \begin{array}{l} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{x} \end{array} \right\} \right] = 0. \tag{E1i}$$

Whereas the determination of most of these expressions is very simple, those containing the  $\text{Li}_2$ -functions are not so easy to handle.

To determine the limit of  $G(x, y)$  in Eq. (5.8f) we have to prove Eqs. (E1g) and (E1h). First, the expression

$$\lim_{\tau \rightarrow 0} \tau^{-1} \left( 2\bar{F}[\ln(y)\ln(\bar{x}) + \text{Li}_2(y) + \text{Li}_2(\bar{x})] - 2F[\ln(\bar{y})\ln(x) + \text{Li}_2(\bar{y}) + \text{Li}_2(x)] \right). \tag{E2}$$

may be simplified. Taking into account  $\text{Li}_2(x) + \text{Li}_2(1-x) = -\ln(x)\ln(1-x) + \pi^2/6$ , we get

$$\begin{aligned}
\ln(y)\ln(\bar{x}) + \text{Li}_2(y) + \text{Li}_2(\bar{x}) & = \frac{\pi^2}{6} + \text{Li}_2(\bar{x}) - \text{Li}_2(\bar{y}) - \ln(y)\ln\left(\frac{\bar{x}}{\bar{y}}\right), \\
\ln(\bar{y})\ln(x) + \text{Li}_2(\bar{y}) + \text{Li}_2(x) & = \frac{\pi^2}{6} + \text{Li}_2(\bar{y}) - \text{Li}_2(\bar{x}) - \ln(x)\ln\left(\frac{\bar{x}}{\bar{y}}\right). \tag{E3}
\end{aligned}$$

From this, together with [33]

$$\begin{aligned}
\text{Li}_2(\tau) & = \tau + \frac{1}{4}\tau^2 + \dots \\
\text{Li}_2\left(1 - \frac{z}{\tau}\right) & = -\text{Li}_2\left(\frac{\tau}{\tau-z}\right) - \frac{1}{2}\ln^2\left(\frac{z-\tau}{\tau}\right) - \frac{\pi^2}{6} \\
& = \frac{\tau}{z} - \frac{1}{2}\ln^2\left(\frac{z-\tau}{\tau}\right) - \frac{\pi^2}{6} + \dots \tag{E4}
\end{aligned}$$

and the corresponding expansion of  $F$  and  $\bar{F}$ , we obtain the result (E1g).

Consider now the term within the second bracket of (5.8f),

$$\begin{aligned}
\theta\left(\frac{\bar{x}}{y}\right)\theta\left(1 - \frac{\bar{x}}{y}\right)\text{sign}(y) & \left[ 2(F - \bar{F})\text{Li}_2\left(1 - \frac{x}{y}\right) + (F - \bar{F})\ln^2(y) - 2F\ln(x)\ln(y) \right. \\
& \quad \left. + 2F\text{Li}_2(\bar{y}) - 2F\text{Li}_2(x) \right] + \left\{ \begin{array}{l} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right\}. \tag{E5}
\end{aligned}$$

According to our procedure we have to calculate

$$E(z) = \lim_{\tau \rightarrow 0} \tau^{-1} \left[ F(x, y) \left( 2\text{Li}_2 \left( 1 - \frac{x}{y} \right) + \ln^2(y) - 2\ln(x)\ln(y) + 2\text{Li}_2(\bar{y}) - 2\text{Li}_2(x) \right. \right. \\ \left. \left. + \ln^2(\bar{y}) + 2\text{Li}_2 \left( 1 - \frac{\bar{x}}{\bar{y}} \right) \right) - \left\{ \begin{array}{l} x \leftrightarrow \bar{x} \\ y \leftrightarrow \bar{y} \end{array} \right\} \right] \Big|_{x=\frac{z}{\tau}, y=\frac{1}{\tau}}. \quad (\text{E6})$$

Here, we apply  $\text{Li}_2(x) + \text{Li}_2(1/x) = -\frac{1}{2}\ln^2(-x) - \pi^2/6$  to express

$$\text{Li}_2 \left( 1 - \frac{x}{y} \right) = -\text{Li}_2 \left( \frac{x}{y} \right) - \ln \left( \frac{x}{y} \right) \ln \left( 1 - \frac{x}{y} \right) + \frac{\pi^2}{6}, \\ \text{Li}_2 \left( 1 - \frac{\bar{x}}{\bar{y}} \right) = -\text{Li}_2 \left( \frac{\bar{y}}{x-y} \right) - \frac{1}{2} \ln^2 \left( \frac{y-x}{\bar{y}} \right) - \frac{\pi^2}{6} \quad (\text{E7})$$

and the relation  $\text{Li}_2(1-x) + \text{Li}_2 \left( 1 - \frac{1}{x} \right) = -\frac{1}{2}\ln^2(x)$  to get

$$\text{Li}_2 \left( 1 - \frac{x}{y} \right) = -\text{Li}_2 \left( \frac{x}{x-y} \right) - \frac{1}{2} \ln^2 \left( 1 - \frac{x}{y} \right). \quad (\text{E8})$$

Then we obtain

$$E(z) = \\ = \lim_{\tau \rightarrow 0} \tau^{-1} \left[ 2F \left( \text{Li}_2(\bar{y}) - \text{Li}_2(x) - \text{Li}_2 \left( \frac{\bar{y}}{x-y} \right) + \text{Li}_2 \left( \frac{x}{x-y} \right) + \ln(y-x) \ln \left( \frac{\bar{y}}{x} \right) \right) \right. \\ \left. - 2\bar{F} \left( \text{Li}_2(x) - \text{Li}_2(\bar{y}) - \text{Li}_2 \left( \frac{\bar{y}}{x-y} \right) + \text{Li}_2 \left( \frac{x}{x-y} \right) + \ln \left( \frac{y-x}{y\bar{x}} \right) \ln \left( \frac{\bar{y}}{x} \right) \right) \right] \Big|_{x=\frac{z}{\tau}, y=\frac{1}{\tau}}. \quad (\text{E9})$$

From this transformed expression we are able to determine (E1h) using (E4) together with

$$\text{Li}_2(x + \epsilon) = \text{Li}_2(x) - \epsilon \ln(1-x)/x + \dots \quad (\text{E10})$$

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